Capital Replacement and the Demand for Clean Technology

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Abstract

I study how capital replacement shapes green technology adoption and innovation. Empirically, industries with more durable capital exhibit weaker green innovation and energy efficiency responses to energy price increases. A calibrated integrated assessment model combining vintage capital and endogenous innovation shows that limited adoption and innovation constrain substitution away from fossil fuels, making near term decarbonization more reliant on reductions in capital. When firms can instead adopt electric machinery, climate policy can effectively redirect investment towards cleaner equipment. Whether capital replacement constrains the green transition therefore depends on the availability of clean technologies and how policy affects equipment demand.

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1 Introduction

The green transition depends on the development and adoption of cleaner technologies to substitute away from fossil fuel. Data suggests that such substitution is limited in the short run (Hassler, Krusell, and Olovsson, 2021). Capital replacement dynamics can explain this pattern. Fossil fuel consuming equipment is highly durable, delaying the adoption of cleaner technologies (IEA, 2020). Slow replacement may also weaken incentives to develop these technologies (Newell, 2008). This creates a tension for climate policy. Discouraging investment in fossil fuel consuming machines is one way to reduce emissions, but lower demand for these machines may further slow the adoption and development of more fuel efficient alternatives. Relying on innovation, however, may deliver emissions reductions too slowly to meet current climate goals.

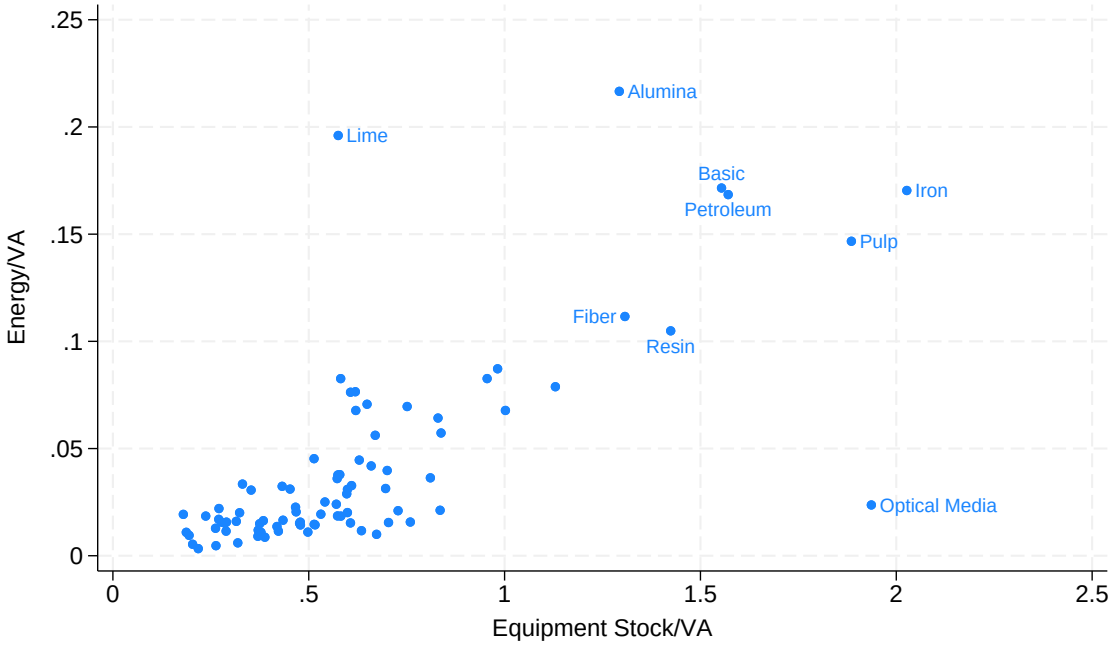
How does the need to replace existing capital affect the adoption and development of clean technologies? Canonical climate economy models with endogenous technical change explain how cleaner technologies are developed but abstract from the capital replacement required to adopt them (Acemoglu, Aghion, Bursztyn, and Hemous, 2012). Putty clay models capture this adoption constraint but do not model innovation (Atkeson and Kehoe, 1999). I study the feedback between capital replacement and energy saving innovation. These feedback effects are important because climate policy often changes the demand for new machines, affecting the incentives for energy saving innovation. The same market size effect that dampens innovation when carbon taxes reduce investment gives replacement subsidies a role in stimulating innovation by expanding demand for new machines.

I combine new reduced form evidence with a structural model to quantify how the interaction of adoption and innovation matters for the green transition. To illustrate the role of durability, I show that industries with high durability exhibit weaker shifts toward clean innovation and smaller improvements in energy intensity in response to energy price shocks. Motivated by this evidence, I develop a multisector integrated assessment model (IAM) with vintage capital and endogenous energy saving technical change. I use the calibrated model to study the mechanics of a green transition in which cleaner technologies must be developed and adopted through capital replacement. Limited adoption and innovation severely constrain substitution toward green technologies, making near-term decarbonization more reliant on reductions in the scale of the economy. I extend the model to allow firms to adopt an existing clean technology, such as electric machinery, rather than rely on endogenous improvements in fuel efficiency. Electrification allows policy to redirect investment toward cleaner equipment through carbon taxes and targeted subsidies. Together, these results indicate that the extent to which adoption presents a bottleneck for the green transition crucially depends on the availability of clean technologies.

To motivate the analysis, I first empirically study how capital durability shapes adjustment to higher energy prices. Figure 1 shows a tight link between capital and energy intensity across U.S. manufacturing industries.[1] If improving energy efficiency requires replacing a large amount of this capital, industries

[1] Manufacturing accounts for 23% of all carbon emissions in the U.S. economy, even before accounting for indirect emissions through the use of electricity generated by electric utilities (EPA, 2021). The majority of these emissions come from energy use.

Figure 1: Capital versus Energy Intensity Across Manufacturing Industries



This figure plots real equipment capital and real energy expenditure, each divided by real value added, across 85 four digit U.S. manufacturing industries in 2000, all in 1997 prices. Energy expenditure and value added are deflated by industry energy and output price indices before aggregation. Equipment capital is already measured in real terms. Only selected industries are labeled. Data are from the NBER CES Manufacturing database.

that naturally replace their capital more often should adjust more quickly. To test this hypothesis, I exploit two long upswings in aggregate energy prices, starting in 1973 and post 2000, each preceded by periods of relative price stability. I use an event study design around these increases to test for differential responses as a function of industry depreciation rates. High depreciation industries experience larger gains in energy efficiency and green patenting. The innovation effects are consistent with greater replacement positively affecting R&D incentives. Quantitatively, industries at the 75th percentile of the depreciation rate distribution experience a 14% decline in energy intensity and an approximately 30% increase in the share of green patents, relative to the 25th percentile.

To interpret the implications of these estimates for the dynamics of the green transition, I develop a theoretical model of adoption and innovation in which demand for new vintages arises from firms replacing old capital with new. The model quantifies (i) long and short run elasticities of energy demand, (ii) the interaction between adoption and technological development, and (iii) transition paths under carbon taxes, replacement subsidies, and industrial electrification.

The key novelty of the model is the two way relationship between capital replacement and energy saving innovation. On the demand side, users of capital goods operate machines with fixed energy requirements, which they can scrap and replace with new machines. The energy efficiency of this new machine depends on the R&D decision of machine producers supplying the capital. Because R&D to improve energy efficiency entails a fixed overhead cost, the model features a market size effect: higher replacement rates increase

innovation incentives. Climate policy therefore affects innovation through both the value of saving energy and the overall investment rate. Carbon taxes increase the value of energy efficiency, a widely studied induced innovation effect, but can also reduce equipment purchases, attenuating an otherwise stronger innovation response. The negative effect on investment arises in the quantitative model because carbon taxes depress the return on investing in a new fossil fuel powered machine.[2] Conversely, investment subsidies can stimulate energy saving innovation by expanding demand for new machines, even without increasing fossil fuel prices.

I embed this model of replacement driven technological change into a dynamic general equilibrium model with climate economy interactions, following Golosov, Hassler, Krusell, and Tsyvinski (2014). Manufacturing sectors differ in capital durability, mirroring the empirical analysis. The calibration combines parameter estimates from external sources with moment matching, targeting sectoral data on investment, depreciation, energy use, and energy prices for the U.S. economy. I calibrate the dispersion of replacement cost shocks to match the differential energy efficiency responses across industries. The calibrated model matches the full time path of the coefficients from the event study. Matching these empirical transition speeds provides credibility to the central goal of my quantitative exercises: How much does slow uptake of new vintages and its feedback effects on innovation matter for short and medium run energy dynamics?

In the first set of counterfactuals, I study how the economy responds to a permanent 150% increase in energy prices.[3] Such an increase is on the same order of magnitude as the mid 1970s run up in energy prices, as well as projections of what is required for the European Union to meet its decarbonization goals (Hochmuth, Krusell, and Mitman, 2025). The simulations indicate a prolonged transition to an equilibrium with higher energy efficiency. For example, in durable manufacturing the transition half life is 28 years for the average technology embodied in firms' capital. These slow improvements in energy efficiency are accompanied by an initial sharp decline in investment, which slows adoption further and lowers innovation incentives. To isolate the role of these market size effects, I introduce a research subsidy that compensates machine producers for lower equipment demand. Offsetting the market size effects in this way raises the cumulative reduction in manufacturing fuel use attributable to innovation by about one third over the first 20 years. These gains are partly offset by a smaller contraction in capital. Thus, lower investment affects both how much fuel use declines and how that decline is achieved.

The importance of these replacement and innovation dynamics depends on how quickly the existing capital stock turns over. I consider a counterfactual economy with depreciation calibrated to match a half life of 5 years for capital goods. In this higher depreciation economy, cumulative manufacturing fuel savings over the first 20 years are about one third greater than in the main model. Three factors account for these differences. First, higher depreciation raises transition speeds towards a lower capital steady state as in the neoclassical growth model. Second, relatively higher adoption leads to more energy efficiency innovation.

[2]The negative effect of carbon prices and fuel costs on investment is in line with well identified macro evidence (Känzig, 2021, forthcoming). However, when I extend the model to include an option to upgrade to electric machinery, fossil fuel investment declines, while investment in electric machinery increases in response to carbon taxes, in line with evidence in Brown, Martinsson, Strömberg, and Thomann (2026).

[3]I treat the price of fossil fuels as exogenous to the U.S. economy.

Third, higher depreciation generates faster adoption of more energy efficient machines. Quantitatively, faster adoption explains the majority of the additional savings, followed by faster capital adjustment and a smaller contribution from greater innovation.[4]

This lack of innovation and adoption also matters for carbon pricing. I compare the effectiveness of optimal carbon taxes in the main model and a Cobb–Douglas benchmark. Optimal taxes are very similar across models under the damage specification in Golosov et al. (2014), but the limited substitutability of the vintage economy leads to smaller emissions reductions in the short and medium run. Matching the medium run emissions reduction of about 47% under Cobb-Douglas requires a constant carbon tax more than four times as high. When I deviate from the Golosov et al. (2014) damage function and assume increasing marginal damages of CO_2, slow replacement leads to higher optimal taxes.[5]

These results highlight the difficulty of a green transition that requires both developing cleaner technologies and replacing existing capital to adopt them. Electrification offers an alternative pathway: firms can switch from fossil fuel to electric machines without waiting for further improvements in fuel saving technology. Electrified alternatives appear to already exist in many industries.[6] Capital replacement remains central to adopting this technology: manufacturing plants report that most natural gas could not be switched to other fuels due to equipment limitations (U.S. Energy Information Administration, 2025). I introduce electrification to the model by giving firms the option to purchase an electric machine when replacing their equipment. The extension shows that the possibility of electrification significantly increases substitution away from fossil fuels under carbon taxes: a permanent $40 carbon tax reduces cumulative manufacturing fuel demand over the first 25 years by 23%, compared with 11% in the main model without electrification.

The distinction between improving fossil fuel machines and switching to electric alternatives also shapes the effectiveness of replacement subsidies. The framework provides a natural setting to assess how such policies affect both adoption and innovation. In the baseline model, broad replacement subsidies stimulate energy saving innovation by expanding demand for new machines. Quantitatively, these improvements are largely offset by scale effects: more frequent replacement raises the economy's capital stock, leaving manufacturing fossil fuel use unchanged. Targeted subsidies can perform better when firms have the option to electrify. An electric machine subsidy with similar fiscal cost reduces manufacturing fossil fuel use by 14% after 25 years. Finally, I compare electrification subsidies and carbon taxes that achieve the same reductions in manufacturing fossil fuel demand after 25 years. While subsidies can attain these reductions, they distort investment choices, yielding lower welfare than carbon taxation. For these

[4]Two extensions highlight the robustness of the adoption channel. First, I consider a calibration with larger intertemporal R&D spillovers to examine whether stronger innovation responses substantially accelerate the transition. Investment declines and slow adoption continue to limit short and medium run substitution. Second, I examine the role of irreversibility in generating possible lock in and slowing the transition. The extension allows firms to resell old capital to a fringe of perfectly competitive firms. While resale changes replacement incentives and allows old machines to remain in use, the resulting path of manufacturing fuel reductions remains quantitatively similar.

[5]These effects are in line with findings by Casey (2023) that models where fossil fuels and other inputs enter in Cobb-Douglas fashion substantially understate carbon taxes needed to reach exogenously specified climate goals.

[6]Comparing U.S. data to Swedish electrification rates reported in Kaartinen and Prane (2024), the electrification gap appears across many 3 digit manufacturing sectors (cf. Figure A8).

manufacturing fuel savings to reduce emissions in the aggregate, the additional electricity must be supplied by a sufficiently clean energy sector. I discuss robustness of these results to assumptions on the carbon intensity of additional electricity supply.

Related Literature. This paper contributes to several strands of the literature on environmental macroeconomics and growth. First, I contribute to the literature using integrated assessment models to study optimal climate policy (Barrage and Nordhaus, 2024; Golosov et al., 2014). These models provide quantitative benchmarks with exogenous technology. Acemoglu et al. (2012) study how policy can redirect innovation toward clean production. Hassler et al. (2021) and Casey (2023) show how energy saving technical change raises long run substitutability between energy and other inputs. My model adds to this literature by quantifying long run substitution through gradual replacement of long lasting capital. Replacement determines both how quickly cleaner technologies are adopted and the demand facing machine producers. This feedback allows policies affecting equipment purchases, such as carbon taxes or investment subsidies, to influence the development as well as the adoption of cleaner machinery.

A smaller literature studies the role of capital replacement for growth and innovation, emphasizing market size effects linked to the replacement rate (Bertolotti and Lanteri, 2024; Hsieh, 2001; Krusell, 1998). Applying this framework to the green transition, Bertolotti, Lanteri, and Yoon (2026) study optimal replacement subsidies that reduce environmental damages and encourage innovation. While they assume an exogenously falling environmental footprint of new machines, my model centers on how energy prices and investment demand affect incentives for energy saving innovation. This allows me to quantify the tradeoff faced by climate policies that generate both market size and substitution effects on innovation. In line with their analysis, I show that replacement subsidies can be an effective second best policy when cleaner alternatives are already available.

An important idea of this paper is that technology is embodied in the capital stock, so benefiting from technology requires investment. This idea is central to "putty clay" models of capital accumulation. Atkeson and Kehoe (1999) were the first to apply the putty clay approach of Solow (1962) to the capital energy nexus. Recent work asks whether sharp increases in carbon taxes or energy prices strand assets by making inefficient vintages unprofitable to operate (Campiglio, Dietz, and Venmans, 2022; Gilchrist, Martinez, and Rickard, 2024; Wei, 2003). My framework abstracts from the short run costs of stranded assets, and instead emphasizes the feedback between innovation and adoption along the transition. This feedback matters quantitatively, as energy prices or carbon taxes encourage innovation, which allows the economy to transition away from fossil fuels. My model of demand driven innovation also accounts for innovation spillovers driven by market size effects. These spillovers are absent in putty clay models where producing firms choose their own capital energy mix when building a new machine. Market size effects provide an important rationale for replacement subsidies, which I analyze in this paper.

On the empirical side, this paper connects to a rich literature emphasizing the importance of innovation and capital adjustment costs in mediating the effects of energy price increases. Hawkins-Pierot and Wagner (2023) show that the energy efficiency of entering firms is significantly more sensitive to rising energy

prices than that of incumbent firms. Relatedly, Capelle, Kirti, Pierri, and Bauer (2023) link the vast heterogeneity in energy intensity across firms in the same industry to the age of the capital stock. While such age driven, within industry dispersion also arises in my model, I emphasize sectoral characteristics shaping the speed at which industries adjust. Finally, my estimates of the differential cross industry effects of energy prices on green innovation contribute to the literature on innovation responses to energy price shocks and environmental regulation (Aghion, Dechezleprêtre, Hemous, Martin, and Van Reenen, 2016; Dugoua and Gerarden, 2025; Popp, 2002). I add to this literature by showing that feedback from adoption to innovation can undermine innovation incentives in low depreciation sectors.

Roadmap. This paper is structured as follows. Section 2 presents the empirical results on the differential effects of energy prices across manufacturing industries differing in the capital depreciation rate. Section 3 develops the general equilibrium model that links capital replacement decisions to energy saving innovation. Section 4 describes the quantitative results on energy price increases, carbon taxes and replacement subsidies. Section 5 concludes.

2 Empirics

The main aim of the empirical analysis is to study the role of slow depreciation and durability as factors mediating the pass through of energy prices to energy efficiency. This section describes construction and sources for the key inputs into this investigation as well as summary statistics. Then I present the results for the main empirical exercise comparing energy efficiency and rates of green patenting across two major episodes of energy price increases.

2.1 Data, Methodology and Descriptives

This section introduces the three main data sources for the empirical analysis. Additional data used for robustness checks is discussed at the end of the results section.

Manufacturing Data. Data on value added, energy expenditure and energy price indices comes from the NBER CES Manufacturing database (Becker, Gray, and Marvakov, 2021). I deflate energy expenditures and value added using industry specific output deflators and industry specific energy price indices. The data on equipment stock shown in Figure 1 uses the real value of the equipment capital stock reported in this database.

Patent data. I start from the universe of USPTO granted patents available from Patstat. I construct annual patent counts from patent families containing a granted U.S. application, assigning each family to its earliest U.S. filing year. In robustness checks, I estimate my regressions on the subset of biadic patents, i.e. patents that were granted by one other patent office in addition to the USPTO. Aggregating this data to the industry level requires a concordance since patent data is reported at the level of technology classes. Let $p_{jk} = 1$ if patent k lists CPC4 technology code j and let N_k be the total number of codes of patent k. A single patent may have several technology codes associated with it. The patent count for

industry i is then constructed as

$$p_i = \sum_j \sum_k \omega_{ij} \frac{p_{jk}}{N_k}$$

where ω_{ij} is concordance weight to aggregate patents from CPC4 to NAICS4 from Lybbert and Zolas (2014). The number of green patents is constructed analogously from all USPTO granted patents that list among their technology code the classification "Y02". This classification has been used widely in academic research (Calel and Dechezleprêtre, 2016; Känzig, forthcoming).

Depreciation data. I construct industry level depreciation rates from equipment capital stock and investment data at the 4 digit NAICS level available from the Federal Reserve. With this data, I estimate depreciation rates by inverting the following perpetual inventory equation

$$K_{i,t+1} = (1 - \delta_{i,t})K_{i,t} + I_{i,t}. \tag{1}$$

This equation gives an implied depreciation rate from the Federal Reserve's equipment capital stock and investment series (Gilbert and Mohr, 1996). These rates vary across years because of shifts in the composition of investment across assets, base year effects (Whelan, 2002), and the cyclical behavior of investment. To smooth this variation, I average the implied depreciation rates from 1991 to 2020, a period during which depreciation rates were relatively constant in the aggregate (Dalgaard and Olsen, 2025). One salient concern is that these measures of depreciation are endogenous to firms' retirement decisions. To assess this concern, I compare industry depreciation rates with average equipment retirement rates in 1987 and 1992 and find a weak, statistically insignificant correlation of 0.1.[7] Appendix A.1 provides further details on the construction of the depreciation measure. I also show robustness to an alternative measure of capital durability based on BLS equipment service lives and BEA investment weights.

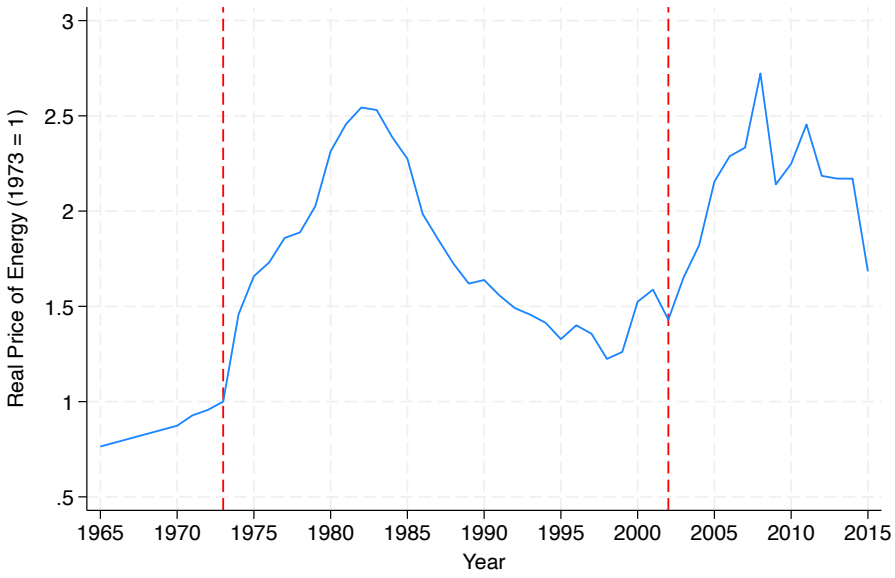
Empirical approach. My analysis aims to test whether pass through of energy price shocks to energy intensity and the share of green patents depends on capital durability. The empirical strategy leverages two sources of energy price variation. First, I evaluate the differential response of industries to aggregate swings in the price of energy using an event study framework. Second, I use an IV approach that relies on industries' differential exposure to fuel price shocks to generate plausibly exogenous variation in energy prices across industries.

Event Studies. Figure 2 plots the evolution of the average price of energy paid by U.S. manufacturing industries.[8] The time series exhibits two distinct upswings. The Panel on the left shows that the real price of energy rises by a factor of 2.5 within 15 years after the 1973 oil price shock. The first oil price shock does not fully explain the persistent increase. Historical accounts of U.S. energy prices emphasize additional domestic and global shocks to energy markets during this time period (Holdren, 1990; Lifset,

[7] Retirement rates are constructed from the publicly available Industry Series for the 1987 and 1992 U.S. Census of Manufactures as equipment retirements divided by the book value of equipment. I average the rates within SIC industries across the two years and map them to NAICS industries using employment weights from the SIC87–NAICS97 crosswalk.

[8] This data is provided by the State Energy Data System (SEDS) as collected by U.S. Energy Information Administration (EIA).

Figure 2: Energy Price Increases in 1970s and 2000s



Note: This plot shows the annual manufacturing energy price calculated as a fuel share weighted average across energy sources from the U.S. Energy Information Administration, deflated by the CPI. Underlying fuel prices are averages for the industrial sector.

2014). An important question is whether industries anticipated such a persistent response to the initial shocks. Kilian and Murphy (2014) argue that inventory behavior indicates producers expected prolonged periods of higher prices after the initial shock, and long run energy price forecasts at the time indicated even more persistent increases (Alpanda and Peralta-Alva, 2010). The Panel on the right shows a smaller but significant run up in energy prices from 2000 to 2005, again led by a dramatic post 2000 increase in oil prices. I use these two episodes of rapid, persistent increases to evaluate how industries respond to such shocks.[9]

Concretely, I estimate the differential response to the shock following the local projections approach to difference in differences estimation suggested in Dube, Girardi, Jorda, and Taylor (2023)

$$\log(y_{it}) - \log(y_{i-1}) = \alpha_t + \beta_t \delta_i + \boldsymbol{\gamma_t}' \boldsymbol{x_i} + \epsilon_{it} \quad \text{for } t \in \{-5, \ldots, 15\} \tag{2}$$

This regression compares how energy intensity and the share of green patents evolve relative to 1973 and 2000, respectively. These reference years correspond to event time $t = -1$, while the first post shock years, 1974 and 2001, correspond to $t = 0$. Coefficients are normalized to zero in the reference year. Coefficients β_t $t < 0$ reveal whether industries trended differentially prior to the increase in energy prices. Because depreciation may correlate with industry characteristics, I control for capital and energy intensity, and fuel shares to capture secular fuel switching trends. Standard errors are clustered at the four digit NAICS level.

[9]While oil is not a major fuel in manufacturing, U.S. fossil fuel prices comove strongly (Serletis and Herbert, 1999), which explains the large energy price increase following the oil shock in Figure 2.

Descriptive Patterns. Table 1 provides summary statistics for the key variables. Panels A and B show these variables for the 5 lowest and highest depreciation rate industries in the sample. These panels illustrate the type of comparisons the empirical analysis relies on. Industries such as steel or industrial machinery feature low depreciation rates around 5 percent, while capital stocks in the apparel industry feature much higher depreciation rates. The variation in depreciation rates closely tracks differences in mean service lives. Capital in low depreciation industries has on average much longer service lives, with a correlation of -0.82 shown in Panel C. By contrast, depreciation is only weakly related to capital intensity, the energy cost share, or the share of green patents. These low correlations indicate that durability and energy intensity are distinct at the industry level. Capital stocks for the higher depreciation industries in Panel B still require significant energy expenditures to operate. Among low depreciation industries (e.g., alumina, steel, machinery manufacturing), energy use varies widely. Likewise, high depreciation industries (e.g., apparel) differ substantially in energy intensity.

Table 1: Comparison of High and Low Depreciation Industries

Industry	Depreciation	Cap. intensity	Energy cost share	Green share	Durability
Panel A: Lowest depreciation					
Steel Product from Purchased Steel	0.05	1.43	0.07	0.06	27.16
Machine Shops, Turned Product, and Screw, Nut, and Bolt	0.05	0.93	0.02	0.02	24.26
Industrial Machinery	0.05	0.86	0.01	0.01	25.21
Coating, Engraving, Heat Treating, and Allied Activities	0.05	0.97	0.06	0.13	24.26
Nonferrous Metal (except Aluminum) Production and Processing	0.06	1.74	0.08	0.08	25.69
Panel B: Highest depreciation					
Veneer, Plywood, and Engineered Wood Product	0.10	0.99	0.09	0.12	15.33
Fiber, Yarn, and Thread Mills	0.10	1.83	0.12	0.02	19.69
Sawmills and Wood Preservation	0.10	0.99	0.07	0.04	15.33
Cut and Sew Apparel	0.10	0.73	0.02	0.01	18.57
Footwear	0.11	0.81	0.01	0.00	18.65
Panel C: Correlations					
Depreciation	1.00				
Cap. intensity	-0.10	1.00			
Energy cost share	-0.09	0.72^{***}	1.00		
Green share	-0.20^{*}	0.16	0.27^{**}	1.00	
Durability	-0.82^{***}	0.35^{***}	0.24^{**}	0.12	1.00

This table presents summary statistics on measures of depreciation, capital intensity, energy intensity, share of green patents and durability across four digit manufacturing industries. All figures present averages using post 1990 data. Industry level depreciation is constructed from inverting the perpetual inventory equation based on industry level capital stocks and investment flows. *Cap. intensity* refers to the ratio of equipment capital to value added. *Energy cost share* is calculated as the ratio of energy costs to value added. *Green share* is the ratio of green to total patents. *Durability* is the weighted average service life of assets using asset industry specific investment as weights. Panels A and B contrast low and high depreciation industries. Panel C shows the correlations between these variables across all 81 manufacturing industries in the sample. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Overall, variation in depreciation rates is sufficiently distinct from plausibly related characteristics identified ex ante, motivating an empirical comparison of high versus low depreciation industries. To ensure robustness to differential pass through by energy or capital intensity, all regressions control for baseline characteristics. I also report estimates without controls.

2.2 Results

Figure 3 shows the results from estimating equation (2) on data pre and post energy price increase. The left hand panel (Figure 3a) shows that after the 1973 oil price shock, energy intensity declines more in

high depreciation industries. The effects are economically sizable: an industry at the 75th percentile of depreciation sees a 14% decline in energy intensity relative to the 25th percentile. The dynamics post energy price shock differ from those observed in the pre period. In times of stable energy prices, high and low depreciation rate industries evolve similarly in terms of energy intensity.

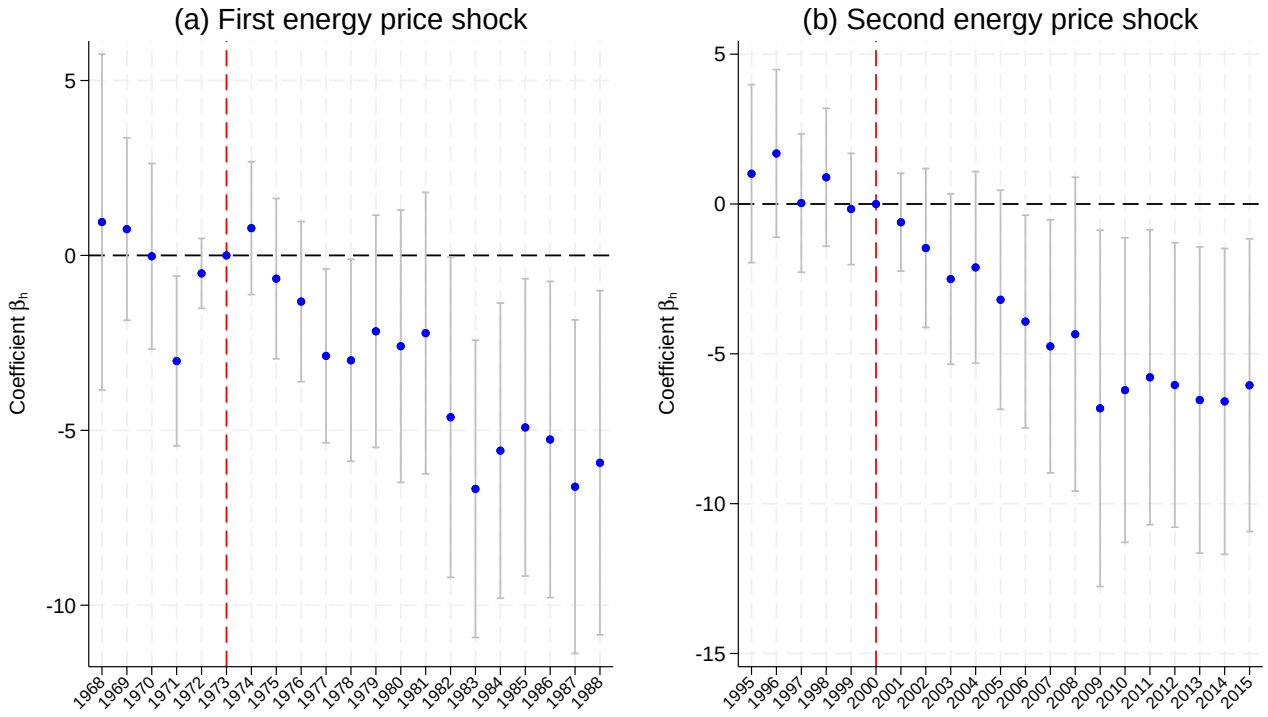


Figure 3: Effect on energy intensity

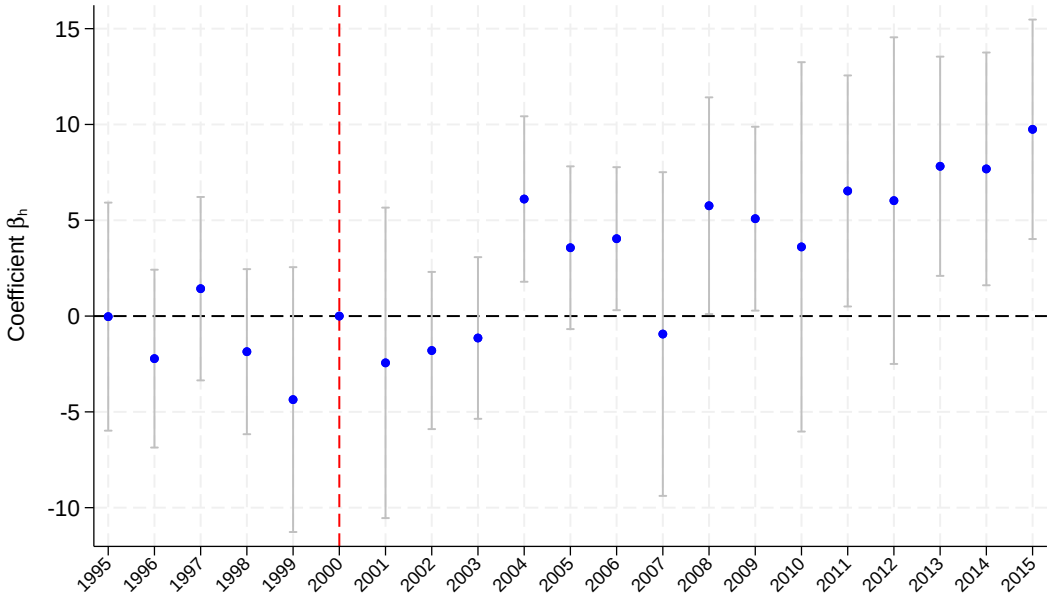
Note: This Figure plots coefficients$\{\beta_t\}_{t=-5}^{15}$ around the two increases in the real price of energy paid by industrial users. The exposure variable to the increase in energy prices is the industry level depreciation rate. The dependent variable is the (log) ratio of real energy use divided by real value added. Panel (a) shows effects around the first energy price increase starting in 1973. Panel (b) repeats shows effects for the second energy price shock starting in 2000. Standard errors are clustered at the four digit NAICS level.

Next, I apply the same approach to the log share of green patents. Since data on USPTO granted patents is available only after 1976, I can only evaluate the effect on the post 2000s run up in energy prices. Figure 4 presents the results. Prior to the energy price shock, there is no differential trend in green patenting. From around 2003, green patenting rises more in high depreciation industries as energy prices increase. By 2015, these estimates imply an approximately 30 percent increase in the share of green patents in industries at the 75th percentile of the depreciation distribution relative to those at the 25th percentile, compared with 2000.

An important question is whether these increases in the share of green patents reflect increases in green patents versus differential declines in overall patents. Results for the (log) level of green and overall patents are presented in Figure A1. Estimates indicate a long run increase in the level of green patents, with a transitory decline in overall patenting rates.

IV estimates. One shortcoming of the event study approach is that major increases in the aggregate

Figure 4: Effect on (log) share of green patents following the energy price increase starting in 2000



Note: This Figure plots coefficients $\{\beta_t\}_{t=-5}^{15}$ around the post 2000 increase in the real price of energy paid by industrial users. The exposure variable to the increase in energy prices is the industry level depreciation rate. The dependent variable is the (log) ratio of green to total patents. Patent data is only available from 1976, so results are only presented for the second energy price increase. Standard errors are clustered at the four digit NAICS level.

price of energy were accompanied by broader macroeconomic downturns. Notably, the response of energy intensity and green patenting does not arise at business cycle frequency. However, short run event study estimates could conflate the reaction to broader macroeconomic events with the reaction to energy prices. To move beyond aggregate increases in energy prices, I rely on industry level energy price indices to test for differential pass through. The regression model is

$$\log y_{it} = \alpha_i + \alpha_t + \rho \log y_{it-1} + \beta_0 \log p^e_{it-1} + \beta_1 \log p^e_{it-1} \times \delta_i + \epsilon_{it}.$$

In this specification, α_i and α_t are unit and time fixed effects. The coefficient of interest is β_1. I include one lag of the dependent variable to allow the effect of energy prices to accumulate over time.[10] Standard errors are clustered at the four digit NAICS level. To address endogeneity concerns about industry level energy prices, I apply an instrumental variables approach. These prices differ because industries differ in the types of fuels they consume (gas, coal, oil, electricity). I construct a fixed weight energy price index:

$$\tilde{p}^e_{i,t} = \sum_f \gamma_{if,1990} p_{f,t}$$

where $\gamma_{if,1990}$ is the share of total energy consumption industry i derived from fuel $f \in \{Coal, Gas, Oil, Electricity\}$ in 1990. $p_{f,t}$ is price observed for those fuels over the period 1990-2018. The data on fuel shares is taken from the Manufacturing Energy Consumption Survey (MECS). The benefit of this IV approach relative

[10] Results are not sensitive to the inclusion of lagged dependent variables.

Table 2: IV estimates: Second stage results

	E/VA			Share Green Patents		
	(1)	(2)	(3)	(4)	(5)	(6)
log(p)	-0.0260	-0.0122	-0.0894	0.346	0.318	-0.0582
	(-0.20)	(-0.09)	(-0.92)	(1.41)	(1.35)	(-0.24)
Lagged Outcome	0.859***	0.855***	0.862***	0.586***	0.570***	0.538***
	(31.61)	(30.15)	(33.44)	(6.46)	(6.26)	(5.52)
log(p) x delta		-2.318**	-2.450***		10.15***	8.585**
		(-2.21)	(-2.70)		(2.80)	(2.33)
Industry F.E.	yes	yes	yes	yes	yes	yes
Year F.E.	yes	yes	yes	yes	yes	yes
Controls	no	no	yes	no	no	yes
Clusters	81	81	81	81	81	81
Kleibergen-Paap F Stat	34.1	18.6	31.6	36.6	19.1	33.8
Observations	2187	2187	2106	2187	2187	2106

t statistics in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Note: This table shows IV estimates of the differential effects of industry level energy prices on the (log) ratio of real energy use divided by real value added (Columns (1)-(3)) and the (log) share of green patents (Columns (4)-(6)). Columns (3) and (6) additionally control for the instrumented interaction of baseline energy intensity and capital intensity with industry level energy prices. First stage estimates are presented in Table A1. Standard errors are clustered at the four digit NAICS level.

to OLS is it avoids reverse causality from reduced energy demand to lower energy prices as well as fuel switching effects. The instruments are relevant, with Kleibergen–Paap statistics ranging from 18.6 to 36.6 across the specifications in Table 2.[11] In addition to a strong first stage, this research design relies on exogeneity of the fuel shares with respect to unobserved shocks to energy intensity and the share of green patents (Goldsmith-Pinkham, Sorkin, and Swift, 2020). In support of this assumption, Figures A2 and A3 show that none of the fuel shares predict changes in the outcomes of interest prior to 1990.

Given these validity checks, Table 2 presents the second stage results. Columns (1)-(3) present results for (log) energy intensity. Column (1) shows that on average energy prices have only a small effect on energy intensity. Column (2) confirms industries with higher depreciation rates see larger reductions in response to increases in energy prices. Column (3) adds control variables by interacting the price of energy with aforementioned industry characteristics. Results remain robust to adding these control variables. The results for the share of green patents paint a similar picture. Column (4) shows a statistically insignificant, positive effect of energy prices on the share of green patents. However the effect on innovation is again larger for high depreciation industries as can be seen in columns (5)-(6). I obtain broadly similar results using OLS instead of the IV approach. Table A2 reports these results.

Robustness Checks. The results are robust along a number of dimensions:

1. Discrete treatment variable: An alternative way of estimating the differential effect of energy prices is to define a dummy variable equal to one if industry i is in the top quartile of the empirical distribution

[11] I have verified that my results also hold in the reduced form, i.e. if I directly plug in $\tilde{p}^e_{it}$ in place of p^e_{it}.

of depreciation rates. Figures A4, A5 and A6 show the results. For the second energy price shock, effects are highly consistent across the two approaches. For the first energy price shock and outcome (log) energy intensity, estimates are qualitatively similar but not statistically significant when using the discrete measure. The depreciation rates are close to uniformly distributed across manufacturing industries such that there is no natural cutoff. Therefore, results based on the continuous measure are arguably preferable.

2. Biadic patents: Patents differ widely in quality, so I repeat the estimation shown in Table A2 including only biadic patents. I define a patent as biadic if it has been granted by USPTO and one other patent agency. Columns (4)-(6) in Table A3 show the results. This delivers very similar results to the OLS estimates of Table A2.
3. Alternative durability measure: To assess the robustness of my results to alternative measures of durability, I construct a measure of average asset life of equipment capital for each industry T_i. This measure is based on BLS estimates of asset lives for disaggregated asset classes and industry by asset class level investment from the 1997 BEA capital flow table. This variable is not always available at the four digit level since BEA industries are often more aggregated than four digit NAICS. The resulting durability measure has a negative correlation with depreciation of -0.82 as shown in Table 1. Table A4 shows the results. While results are slightly noisier, they remain statistically significant once controls are included.
4. Equipment purchases: The proposed mechanism operates through demand for new machines. To examine this channel more directly, I study equipment purchases at the asset level, relating their response to energy prices to the service life of the equipment being purchased. I combine BEA data on equipment purchases by manufacturing firms with the same BLS service lives underlying the alternative durability measure. Figure A7 shows that purchases of longer lived equipment fall relative to those of shorter lived equipment after both energy price increases. This pattern is consistent with smaller contractions in demand for less durable equipment sustaining a larger market for new machines and supporting innovation incentives.

Summary. These results show that industries with higher depreciation rates experience larger improvements in energy intensity and green patenting after energy prices increase. The findings are consistent with capital replacement facilitating the adoption of more efficient equipment and increasing incentives to develop cleaner machines. The model examines this mechanism and its implications for the green transition.

3 The Model

Model overview. The model in this section describes the feedback between incentives of machine users to replace capital and suppliers of the capital to improve the energy efficiency of those machines. Machine users differ across and within sectors. Across sectors, machines depreciate more slowly in the sector with

more durable capital. This sectoral heterogeneity mirrors the reduced form cross industry regressions. Within sectors, firms receive idiosyncratic shocks to the cost of replacing their capital. Firms adopt new technology only when they replace their machines, so differences in replacement decisions generate a distribution of vintages in each sector.

Faced with this demand side, innovating firms choose price and energy efficiency embodied in these machines. The model features a market size effect whereby higher investment rates increase innovation because it allows firms to spread the overhead cost of R&D over more units. Higher energy prices or more stringent carbon policy makes energy efficient machines more valuable, but can also reduce equipment purchases and thereby weaken innovation incentives.

I embed this two way relationship between adoption and innovation in a standard macro climate general equilibrium model following Golosov et al. (2014). The model quantifies how slow machine replacement shapes the effects of energy price shocks, carbon policy, and the benefits of policies targeting investment.

3.1 Growth model

I consider a continuous time, infinite horizon economy. The preferences of the infinitely lived, representative household are represented by the utility function

$$U = \int_0^\infty e^{-\rho t} \log C(t) dt \tag{3}$$

where $C(t)$ is consumption of the final good and ρ is the discount rate. The representative household saves in bonds at interest rate $r(t)$, leading to a standard Euler equation for consumption growth

$$\frac{\partial C(t)}{\partial t} = C(t)(r(t) - \rho). \tag{4}$$

The final output good is produced by combining manufacturing output, X_m, services X_s and electricity X_e

$$Y(t) = \left((X_m(t)^\kappa X_s(t)^{1-\kappa})^{\frac{\varrho-1}{\varrho}} + \nu X_e(t)^{\frac{\varrho-1}{\varrho}} \right)^{\frac{\varrho}{\varrho-1}}. \tag{5}$$

Constant normalizations of the sectoral technologies are absorbed into productivity A_L.

Denote output net of climate damages by $\widetilde{Y}(t) = e^{-\psi(S(t)-\bar{S})} Y(t)$. The term $e^{-\psi(S(t)-\bar{S})}$ expresses aggregate productivity as a function of climate damages as in Golosov et al. (2014). $S(t)$ denotes the amount of carbon in the atmosphere in period t, equal to $\bar{S}$ in the base period. The dynamics of the climate system are represented as in Golosov et al. (2014):

$$S(t) = S_P(t) + S_D(t), \tag{6}$$

$$\dot{S}_P(t) = \varphi_L \, \mathcal{E}(t), \tag{7}$$

$$\dot{S}_D(t) = -\varphi_S \, S_D(t) + \left(1 - \varphi_L\right) \varphi_0 \, \mathcal{E}(t). \tag{8}$$

Atmospheric carbon stock $S(t)$ is the sum of carbon that remains in the atmosphere permanently $S_P(t)$ and carbon that is partially absorbed by the climate system over time $S_D(t)$. A share φ_L of current emissions $\mathcal{E}(t)$ stays in the atmosphere forever, increasing $S_P(t)$. φ_0 is the share of nonpermanent emissions that is not immediately absorbed, while φ_S determines the rate at which nonpermanent carbon is absorbed over time.[12]

The three inputs X_i into this aggregate are produced as follows. Services are produced linearly from labor $X_s(t) = L_s(t)$. Manufacturing combines capital and labor using a Cobb-Douglas production function

$$X_m(t) = K_m(t)^{\alpha} L_m(t)^{1-\alpha}, \tag{9}$$

where

$$K_m(t) = \left[\beta^{1/\eta} K_d^u(t)^{(\eta-1)/\eta} + (1-\beta)^{1/\eta} K_{nd}^u(t)^{(\eta-1)/\eta}\right]^{\eta/(\eta-1)}. \tag{10}$$

Here $K_i^u(t)$ denotes utilized capital in manufacturing sector i, defined below.

The two sectors divide manufacturing into one producing with more versus less durable capital mirroring the reduced form analysis. Finally, electricity production combines electricity capital and labor

$$X_e(t) = K_e(t)^{\varphi} (A_L L_e(t))^{1-\varphi}. \tag{11}$$

Electricity capital $K_e(t)$ is a composite of fossil fuel capital $K_e(f,t)$ and green energy capital $K_e(g,t)$

$$K_e(t) = \left(K_e(f,t)^{\eta_e} + A_g^{1/d_e} K_e(g,t)^{\eta_e}\right)^{1/\eta_e}. \tag{12}$$

Here $\rho_e = (\vartheta - 1)/\vartheta$, $d_e = 1 - (1-\varphi)\rho_e$ and $\eta_e = \varphi\rho_e/d_e$. The parameter ϑ is the elasticity of substitution between clean and fossil fuel electricity outputs. Appendix B.2 derives this capital aggregate after allocating labor between the clean and fossil fuel using subsectors. Clean capital operates without fossil fuel, while fossil fuel capital requires $1/a_f$ units of fuel per unit of capital services.

3.1.1 Investment dynamics

I now describe the replacement problem of manufacturing firms. Each sector $i \in \{d, nd\}$ contains a fixed mass N_i of machine users. I suppress the sector index when describing their decisions. Each user operates a bundle of machine varieties, which it replaces together. Electricity producers instead choose investment in homogeneous capital, as described below.

Static Profit Maximization. I first describe the static profit maximization problem for firms in each sector and then determine their investment policy. A machine user produces capital services $y(t)$

[12] The Golosov et al. (2014) climate model provides a strongly simplified representation of the climate system relative to more full fledged models (Folini, Friedl, Kübler, and Scheidegger, 2025). Importantly, Dietz, Van Der Ploeg, Rezai, and Venmans (2021) show that this simple representation accurately captures the immediate link between cumulative emissions and surface warming. The accurate representation of near term climate dynamics makes it a useful benchmark for examining how short run technological lock in alters carbon policy conclusions. A shortcoming of this climate model is that it abstracts from state dependence in terms of the climate system's ability to absorb carbon over time (Dietz et al., 2021).

from a continuum of varieties $\nu \in [0,1]$:

$$y(t) = \left[\int_0^1 \min\{A_e(\nu, t-a)e(\nu,t), k(\nu,t)\}^{(\sigma-1)/\sigma} \, d\nu \right]^{\sigma/(\sigma-1)}.$$

Each variety combines capital $k(\nu,t)$ with fuel $e(\nu,t)$. Its energy efficiency $A_e(\nu, t-a)$ is determined at installation. The continuum of varieties makes demand for an individual producer's machines smooth, while the user's decision to replace the entire bundle remains discrete. This production function imposes two important assumptions.

First, firms produce using vintage capital. Denoting by a the age of the machine and by $t-a$ the date of purchase, energy efficiency remains constant across the life of a machine. I further assume that firms can only replace all machines ν at once, implying that $t-a$ does not vary within the firm. A new machine contains $\bar{k}$ units of capital, which depreciate at rate δ.

Second, capital and energy are modeled as perfect complements. Given predetermined capital and technology per machine, profits depend only on utilization, which is determined by the energy input. Faced with an exogenous energy price $p_e(t)$ and taking the output price $P^K(t)$ as given, firms solve:

$$\max_{\{e(\nu,t)\}} P^K(t)y(t) - p_e(t) \int_0^1 e(\nu,t)\, d\nu, \qquad 0 \leq e(\nu,t) \leq \frac{k(\nu,t)}{A_e(\nu, t-a)}$$

and the resulting energy use satisfies

$$e(\nu,t) = \min\left\{ y(t) \left(\frac{P^K(t)}{p_e(t)} \right)^{\sigma} A_e(\nu, t-a)^{\sigma-1}, \; \frac{k(\nu,t)}{A_e(\nu,t-a)} \right\}.$$

In equilibrium, the allocation of capital across machines and their energy efficiency are symmetric: $k(\nu,t) = k(t)$ and $A_e(\nu,t-a) = A_e(t-a)$. In that case, profits can be expressed in terms of a common energy efficiency $A_e(t-a)$, the firm's capital $k(t)$ and its utilization u

$$\pi(k,t) = \max_{u \in [0,1]} uk\bar{\pi}(k,t).$$

Profits are equal to the level of capital $k(t)$ multiplied by its marginal revenue product $\bar{\pi}(k,t) = \left(P^K(t) - \frac{p_e(t)}{A_e(t-a)} \right)$ which is increasing in the output price $P^K(t)$ and the level of technology $A_e(t-a)$ and declining in the energy price $p_e(t)$. Whenever marginal revenue is negative, the firm does not utilize its capital $u = 0$ and profits are zero. Profits do not differ across varieties within a firm, but they do differ across firms that replaced capital at different times $t-a$ due to differences in the technology embodied in their capital $A_e(t-a)$.

Discussion of assumptions. Before describing the role of this profit function for investment dynamics, I briefly discuss the assumptions underlying it. Prior research documents the empirical relevance of technological lock in for understanding energy efficiency dynamics in manufacturing. Manufacturing plants are most sensitive to energy prices at entry (Hawkins-Pierot and Wagner, 2023), consistent with

technological lock in after major investments. Linn (2008) documents higher energy efficiency among entrants than incumbents, suggesting that firms benefit from technology growth when they invest in new capital. Second, I assume capital and energy are perfect complements within each machine. At the firm level, several papers estimate a very low elasticity of substitution between energy and nonenergy inputs (Chen, Chen, Liu, Suárez Serrato, and Xu, 2025; Hawkins-Pierot and Wagner, 2023; Ryan, 2018). Hassler et al. (2021) document a near perfect short term correlation between the price of energy and the energy factor share of GDP.

Dynamic Replacement Problem. Firm capital $k(t)$ is determined through irreversible investment in new machines. The investment decision is based on the Hamilton-Jacobi-Bellman equation

$$r(t)V(k,t) = \underbrace{\pi(k,t)}_{\text{Profits}} + \underbrace{V_t - \delta k V_k}_{\text{Drift}} + \underbrace{\lambda(\mathcal{V}(k,t) - V(k,t))}_{\text{Expected Replacement Gain}} . \tag{13}$$

Time t and capital k are the state variables of the firm's problem. Firms also differ in technology, but using $\frac{\partial k(t)}{\partial t} = -\delta k(t)$ we can infer age $a = \frac{1}{\delta}\log\left(\frac{\bar{k}}{k}\right)$ and thus $A_e(t-a) = A_e(k,t)$. $\bar{k}$ denotes the level of capital of a new machine.

The instantaneous return to capital $r(t)V(k,t)$ has three components. First, the flow of profits the firm makes from its capital stock $\pi(k,t)$. The drift component captures deterministic capital gain dynamics due to depreciation, $\delta k V_k$, and potential transition dynamics V_t. Third, the firm receives replacement opportunities at Poisson rate λ. If it receives the opportunity to replace its capital, the firms draws an additive shock ϵ to the value of a new machine $V(\bar{k},t) - P(t)\bar{k}$, yielding an expected gain $\mathcal{V}(k,t) - V(k,t)$.[13] Replacement takes place whenever

$$V(\bar{k},t) - P(t)\bar{k} + \epsilon > V(k,t). \tag{14}$$

Conditional on the shock ϵ, firms are more likely to replace when the purchase price of new capital is low or new machines are more profitable. The shocks can represent idiosyncratic differences in firms' costs to switch from old to new machines. The endogenous cost of new capital $P(t)\bar{k}$ is derived below from innovating firms supplying the capital. Relative profitability of new versus old capital depends on how far the old machine has depreciated, as well as the gap in marginal products. Using the firm-level, discrete choice policy function and integrating over shocks, the expected gross gain is $\mathcal{V}(k,t) = \mathbb{E}_\epsilon\big[\max V(\bar{k},t) - P(t)\bar{k} + \epsilon, V(k,t)\big]$.

Law of motion for the distribution. Let $\mu(k,t)$ be the distribution of capital across firms, with

[13] The assumption of exogenous arrival rate of replacement opportunities with shocks to each option drawn upon arrival follows Arcidiacono, Bayer, Blevins, and Ellickson (2016). An alternative approach would be to continuously allow firms to replace their capital. Introducing idiosyncratic shocks to the resulting HJB Variational Inequality requires adding a second state variable. For details see Achdou, Han, Lasry, Lions, and Moll (2022) and related notes available on Ben Moll's website `https://benjaminmoll.com/codes/`. The current approach avoids this issue because the HJB equation is an equality, allowing me to integrate out the shocks in the expected replacement value.

$\int_0^{\bar{k}} \mu(k,t)\,dk = N$. The distribution evolves according to

$$\partial_t \mu(k,t) = \partial_k[\delta k \mu(k,t)] - \lambda P^R(k,t)\mu(k,t). \tag{15}$$

The first term reflects inflow of firms into capital level k, while the second term reflects outflow of all firms with capital k that choose to replace when faced with the opportunity.[14] The replacement probability $P^R(k,t) = \mathbb{P}(V(\bar{k},t) - P(t)\bar{k} - V(k,t) > -\epsilon)$ is obtained by integrating the investment policy (14) across the density of ϵ. These firms reenter with capital $\bar{k}$, which leads to the boundary condition

$$\delta\bar{k}\mu(\bar{k},t) = M(t), \qquad M(t) = \int_0^{\bar{k}} \lambda P^R(k,t)\mu(k,t)\,dk.$$

Gross equipment purchases are $I(t) = \bar{k}M(t)$. The law of motion for capital $K(t)$ in each manufacturing sector is

$$\frac{\partial K(t)}{\partial t} = I(t) - \delta K(t) - \lambda \int_0^{\bar{k}} P^R(k,t)k\mu(k,t)\,dk.$$

Replacement implies scrapping the remaining capital in old machines and purchasing new machines. In steady state, gross investment replaces capital lost through physical depreciation and scrappage. The investment to capital ratio therefore exceeds physical depreciation. The calibration uses this identity to match the empirically constructed implied depreciation rates, which capture both scrappage and physical depreciation. The sectoral capital aggregates entering the manufacturing production function in equation (10) are restricted to the part of sectoral capital that is utilized.

Demand for new technologies. When a firm receives a replacement opportunity, it can scrap its existing capital and purchase a new machine composed of varieties $k(\nu,t)$. Firms selling the varieties $k(\nu,t)$ also choose their energy efficiency $A_e(\nu,t)$. Thus, the demand for varieties $k(\nu,t)$ determines innovation incentives.

To simplify the analysis, I assume that firms purchase machines anticipating full utilization until replacement. All quantitative exercises verify that this condition holds for the energy price increases and carbon taxes I consider. If unexpected changes in energy prices at $t = 0$ make existing machines unprofitable to operate, firms can leave them idle. With this assumption, we can state the profit maximization problem for individual varieties as

[14] To see the intuition for the capital drift in this equation, abstract from replacement and write the share of firms with capital in the interval $[k,\ k + dk]$ at time $t + dt$ as

$$\mu(k, t + dt)\,dk \ = \ \mu\big(k(1 + \delta dt), t\big)\,(1 + \delta dt)\,dk.$$

The additional factor $(1 + \delta dt)$ reflects the fact that these firms come from an interval of width $dk(1 + \delta dt)$. Taking a first order approximation of $\mu\big(k(1 + \delta dt), t\big) = \mu(k,t) + \delta k \mu_k(k,t)dt$ and letting $dt \to 0$ gives the result. Equivalently, we can derive the law of motion starting from the discrete time law of motion for the cumulative distribution function. I include this derivation in Appendix B.1.

$$\max_{k(\nu,t)} \mathcal{L} = \underbrace{\int_t^\infty e^{-H(t,\tau)}\Big[P^K(\tau)\big(\int_0^1 k(\nu,t)^{\frac{\sigma-1}{\sigma}}\,d\nu\big)^{\frac{\sigma}{\sigma-1}} - \int_0^1 \tfrac{p_e(\tau)}{A_e(\nu,t)}\,k(\nu,t)\,d\nu\Big]d\tau}_{\text{Operating Profits}} - \underbrace{\int_0^1 p(\nu,t)\,k(\nu,t)\,d\nu}_{\text{Investment Cost}}$$
$$+\ \xi(t)\underbrace{\Big[\bar{k} - \big(\int_0^1 k(\nu,t)^{\frac{\sigma-1}{\sigma}}\,d\nu\big)^{\frac{\sigma}{\sigma-1}}\Big]}_{\text{Capacity Constraint}}. \tag{16}$$

Profits stem from operating profits over the replacement cycle net of investment cost. In the presence of perfect competition and constant returns to scale in production, the capacity constraint $\bar{k}$ pins down the overall level of capital.

The length of the replacement cycle is stochastic as reflected by the discount factor

$$H(t,\tau) = \delta(\tau - t) + \int_t^\tau r(u) + \lambda P^R(\bar{k}e^{-\delta(u-t)}, u)du,$$

summarizing depreciation, standard discounting of future profits at the interest rate and the evolution of the replacement probability due to capital depreciation and transition dynamics. By taking into account how this probability changes over time, firms' sensitivity to energy prices crucially depends on future replacement dynamics. Suppose the firm anticipates likely replacement in the future, for example because of higher energy efficiency in the future. The firm then discounts current and future energy prices when buying a machine today. Intuitively, current and future energy prices matter less for today's purchase if the firm anticipates facing those prices with a different machine than the one it is buying today.

As shown in Appendix B.2, this profit maximization problem gives rise to isoelastic demand for machines

$$k(\nu,t) = \bar{k}\left(\frac{\tilde{p}(\nu,t)}{\tilde{P}(t)}\right)^{-\sigma} \tag{17}$$

where $\tilde{p}(\nu,t) = p(\nu,t) + \tilde{p}_e(\nu,t)$ is the sum of the upfront purchasing price $p(\nu,t)$ and life time energy costs given by

$$\tilde{p}_e(\nu,t) = \frac{\tilde{p}_e(t)}{A_e(\nu,t)} = \frac{\int_t^\infty e^{-H(t,\tau)}p_e(\tau)d\tau}{A_e(\nu,t)}.$$

$\tilde{P}(t) = (\int_0^1 \tilde{p}(\nu,t)^{1-\sigma}d\nu)^{\frac{1}{1-\sigma}}$ is the associated CES price index. Demand for machines is influenced both by the purchase price $p(\nu,t)$ and as the downstream energy cost of the machine $\tilde{p}_e(\nu,t)$.

3.1.2 Supply of Technology.

Each machine $k(\nu,t)$ is produced by a monopolistically competitive machine producer choosing price $p(\nu,t)$ and energy efficiency $A_e(\nu,t)$ to maximize profits

$$\pi(\nu,t) = \max_{p(\nu,t),\ A_e(\nu,t)} M(t)k(\nu,t)(p(\nu,t) - c) - \gamma\frac{A_e(\nu,t)^{\frac{1}{\theta}}}{\phi(t)} \ \text{ s.t. } (17). \tag{18}$$

Firms' unit margin $p(\nu,t)-c$ is the difference between the purchase price of a machine and the marginal cost of capital c denoted in units of the final good. Quantity sold is the product of the mass of firms that replace $M(t)$ times quantity sold to each firm that replaces. R&D into energy efficiency represents an overhead cost and is subject to decreasing returns $\theta<1$. Total R&D costs are $\gamma\frac{A_e(\nu,t)^{\frac{1}{\theta}}}{\phi(t)}$ where γ is an exogenous research cost shifter and $\phi(t)$ evolves with cumulative R&D efforts. The optimal price equals

$$p(\nu,t)=\frac{\sigma}{\sigma-1}c+\frac{1}{\sigma-1}\frac{\tilde{p}_e(t)}{A_e(\nu,t)}. \tag{19}$$

A simple intuition for the effect of life time energy costs on the purchase price of a machine comes from the derived elasticity of demand

$$-\frac{d\log k(\nu,t)}{d\log p(\nu,t)}=\sigma\frac{p(\nu,t)}{\tilde{p}(\nu,t)}<\sigma.$$

With higher energy prices, firms become less sensitive to the purchase price of a machine $p(\nu,t)$, which raises markups. In a symmetric equilibrium (cf. Appendix B.2), the first order condition for energy efficiency can be rearranged to

$$A_e(\nu,t)=A_e(t)=(\theta\phi(t)M(t)\bar{k}\tilde{p}_e(t)/\gamma)^{\frac{\theta}{1+\theta}}. \tag{20}$$

Higher energy prices have direct and indirect effects on the supply of energy efficiency. The direct effect stems from higher energy prices increasing capital users' willingness to pay for energy efficiency. Faced with this higher willingness to pay, firms supplying the capital increase energy efficiency as machine users are now more easily induced to buy from them. Indirect effects arise from the effect of energy prices on the replacement rate $M(t)$, depending on whether total fuel costs go up more for old or new machines. If energy efficiency does not increase quickly, total fuel costs will go up more for undepreciated new machines. Faced with relatively lower profitability, firms delay replacement, shrinking the market for new machines and hence equilibrium energy efficiency. Combining (19) and (20), we obtain the investment cost of a new machine

$$P(t)=\frac{\sigma}{\sigma-1}c+\frac{1}{\sigma-1}\frac{\tilde{p}_e(t)}{A_e(t)}. \tag{21}$$

To capture spillovers from innovation, I assume that $\phi(t)$ follows a simple law of motion

$$\frac{\partial\phi(t)}{\partial t}=-\delta_A\phi(t)+A_e(t). \tag{22}$$

Past innovation raises the common knowledge stock, lowering the cost of subsequent improvements in energy efficiency. Firms do not internalize these spillovers because they solve a static profit maximization problem and take research productivity as given. Conditional on $\phi(t)$, producers must undertake research each period to maintain a given level of new machine efficiency. In steady state, $\phi=A_e/\delta_A$, which allows me to characterize energy efficiency analytically.

Equilibrium. Manufacturing capital, utilized capital, and fuel use are

$$K_i(t) = \int_0^{\bar{k}} k\mu_i(k,t)\,dk, \qquad K_i^u(t) = \int_0^{\bar{k}} u_i(k,t)k\mu_i(k,t)\,dk, \qquad E_i(t) = \int_0^{\bar{k}} \frac{u_i(k,t)k}{A_i(k,t)}\mu_i(k,t)\,dk.$$

Here $A_i(k,t)$ is the energy efficiency embodied in a machine with capital k in sector i. The price of utilized capital equals its marginal product, $P_i^K(t) = \partial\widetilde{Y}(t)/\partial K_i^u(t)$.

Green and fossil fuel capital employed in electricity generation evolve as $\frac{\partial K_j(t)}{\partial t} = I_j(t) - \delta_j K_j(t)$ for $j \in \{f, g\}$. The net rental return satisfies $\mathcal{R}_j(t) = c_j[r(t) + \delta_j]$. In fossil fuel electricity these returns are equal to the marginal product of capital net of fuel cost $\mathcal{R}_f = P_f^K - p_{e,f}/a_f$. Clean generation faces no additional fuel cost $\mathcal{R}_g = P_g^K$. Fossil fuel use in electricity generation is $E_f(t) = K_f^u(t)/a_f$, where $K_f^u(t)$ denotes capital in operation. Total carbon emissions are

$$\mathcal{E}(t) = \sum_{i\in\{d,nd,f\}} \chi_i E_i(t) + \mathcal{E}^{\text{exo}}(t),$$

where χ_i converts sectoral fuel use into carbon emissions measured in GtC per year. $\mathcal{E}^{\text{exo}}(t)$ denote emissions outside the model which are held constant across counterfactuals.

Denoting sectoral research expenditure by $R_i(t) = \gamma_i A_{e,i}(t)^{1/\theta}/\phi_i(t)$ and replacement cost shocks from exercised replacement opportunities $B_\epsilon(t)$, we can write the economy's resource constraint as

$$\widetilde{Y}(t) + B_\epsilon(t) = C(t) + \sum_{i=d,nd} [c_i I_i(t) + R_i(t)] + \sum_{j=f,g} c_j I_j(t) + \sum_{i=d,nd,f} p_{e,i}(t)E_i(t), \tag{23}$$

where $p_{e,i}(t)$ denotes the resource cost of fuel, and

$$B_\epsilon(t) = \sum_{i=d,nd} \lambda \int_0^{\bar{k}} \mu_i(k,t)\,\mathbb{E}[\epsilon\,\mathbf{1}\{V_i(\bar{k},t) - P_i(t)\bar{k} - V_i(k,t) + \epsilon > 0\}]\,dk$$

is the resource gain from cost shocks received by firms replacing their capital.

The labor market clearing condition states that perfectly mobile labor is split between manufacturing $L_m(t)$, services $L_s(t)$, and electricity generation $L_e(t) = L_f(t) + L_g(t)$:

$$L = L_m(t) + L_s(t) + L_e(t). \tag{24}$$

3.2 Model Solution and Calibration.

Solution method.

I first solve the economic steady state holding climate damages fixed. Climate stocks then evolve in the policy transitions. The steady state of the model is solved following Achdou et al. (2022). I use a finite difference method to discretize the capital grid and solve the Hamilton-Jacobi-Bellman equation (13) numerically, given a guess of the endogenous variables. To solve for the distribution (15), I apply

this discretization to approximate the derivative $\frac{\partial \mu(k,t)}{\partial k}$ and calculate the model implied replacement probability based on the solution to the HJB. The steady state distribution (15) can then be solved as a linear system.

To compute transition dynamics, I represent aggregate paths on a grid of 19 Chebyshev nodes (Hémous, Lepot, Sampson, and Schärer, 2023; Schesch, 2024). Given these paths, I solve the HJB backward and the distribution forward on finer time grids. I then update aggregate prices, consumption, and technology until the equilibrium conditions hold. Appendix C describes the discretization and terminal conditions used in the quantitative exercises.

Calibration. I calibrate the model to cross sectional moments from the U.S. manufacturing sector for the year 2010 as well as the difference in differences estimates presented in Section 2. Several key parameters are internally calibrated using moment matching. The remaining parameters are set based on external sources. Table 3 lists all parameters, their values and sources used in the calibration.

Dividing manufacturing into two sectors. The model features industries using more and less durable capital. I sort the 86 four digit industries in the NBER CES Manufacturing database based on the depreciation rates used in the empirical analysis. Industries in the lower quartile of the distribution of depreciation rates are sorted into the durable using sector. I use this 25%–75% split to isolate a group of industries with particularly durable capital.

Externally calibrated parameters: climate. I set the permanent share of emissions to $\varphi_L = 0.2$. Of the remaining emissions, a share $\varphi_0 = 0.4$ initially remains in the atmosphere and 60% is absorbed immediately. The decaying carbon stock is absorbed at rate $\varphi_S = 0.00231$. I set the semielasticity of productivity with respect to atmospheric carbon to $\psi = 0.00011$, following the higher value proposed by Acemoglu, Aghion, Barrage, and Hémous (2023). This value is motivated by recent estimates of large GDP losses from warming (Bilal and Stock, 2025; Nath, Ramey, and Klenow, 2025). Initial carbon stocks are $S_P(0) = 684$ GtC and $S_D(0) = 159$ GtC.

Externally calibrated parameters: production. I set the manufacturing capital exponent to $\alpha = 0.5$, implying a labor share of 0.5, consistent with Kehrig and Vincent (2021). BLS data indicate a labor share of 0.25 for electric utilities, so I set $\varphi = 0.75$. I use $\eta = 1.2$ for substitution between manufacturing capital types within the range of cross sector elasticities estimated by Redding and Weinstein (2024) and Oberfield and Raval (2021). Following Papageorgiou, Saam, and Schulte (2017), I set the elasticity between clean and fossil fuel electricity outputs to $\vartheta = 1.85$. The durable capital weight is $\beta = 0.286$, based on the durable share of manufacturing value added.

Externally calibrated parameters - Supply of Technology. I set the elasticity of substitution σ across capital varieties equal to 4. I set $\theta = 0.35$ to allow for substantial diminishing returns to research, as in Acemoglu, Akcigit, Hanley, and Kerr (2016). The common knowledge stock captures research spillovers of the kind documented by Bloom, Schankerman, and Van Reenen (2013). While not directly targeted, this level of decreasing returns is in line with data on energy efficiency R&D. Expenditure on this category of R&D amounts to 5.4% as a share of manufacturing investment in the model, about 72% of

the corresponding share for company funded energy related R&D in the data, a reasonably close fit given that manufacturers also develop products used outside manufacturing, such as cars.[15] To calibrate the law of motion for research productivity $\phi(t)$ in (22), I assume productivity depreciates at rate $\delta_A = 0.05$. This value is consistent with estimates of organizational forgetting for durable goods manufacturing by Benkard (2000) and on the lower end of R&D depreciation values in Hall (2007) for Compustat firms as a whole.

Normalized Parameters. I normalize the size of machines in both sectors to $\bar{k} = 1$ and set the arrival rate of replacement opportunities $\lambda = 1$ as in Arcidiacono et al. (2016). With $\lambda = 1$, firms receive an average of one replacement opportunity per year. Aggregate labor is normalized to ten, meaning the labor productivity parameter should be interpreted in per capita terms. I also set $c = 1$, i.e. a unitary conversion rate of final output to investment as is standard in neoclassical growth.

Internally calibrated parameters. I calibrate labor productivity A_L to match the value of world GDP reported by the World Bank ($66 trillion).[16]

The mass of firms N_m in manufacturing is chosen to match sectoral private investment in fixed assets reported in BEA data (2.4% of GDP). I assume an equal number of firms in each subsector. Physical depreciation rates are chosen to match the perpetual inventory implied rates used in the reduced form analysis based on NBER manufacturing data. I match the observed sectoral depreciation rates of 5.8% and 8.3%. Since empirical depreciation rates are inflated by firms discarding capital before full depreciation (obsolescence), physical depreciation rates are lower. Depreciation rates for fossil fuel and clean capital follow Gilchrist et al. (2024) who set them to 5 and 10 percent, respectively.

I assume shocks to the cost of replacing a machine follow a logistic distribution. I calibrate the scale parameter of the distribution by matching the event study after the 1973 energy price shock. Specifically, I treat the DiD effect in Figure 3a as reflecting the differential impact of a permanent 150% increase in energy prices on average energy efficiency. The DiD identifies the dispersion within this model because with low variance, the decline in investment is amplified which lowers the energy efficiency response.

The exogenous component of R&D productivity γ is set to match sectoral data on energy expenditures relative to value added in the manufacturing sectors. With lower R&D productivity, energy efficiency falls, raising the energy share. Electricity productivity ν is chosen to match the GDP share of electricity (1.86%). The clean energy weight A_g is calibrated to match a clean energy share of 10% based on 2010 data for the U.S.

[15] The NSF/NCSES and Census Bureau's 2018 Business Research and Development Survey reports $19.4 billion in company funded energy related R&D performed by manufacturing firms. The Census Bureau's 2019 Annual Capital Expenditures Survey summary reports 2018 manufacturing investment of $258.1 billion, implying a research expenditure share of 7.5%. Since this data does not separately identify the share of R&D devoted to improving energy efficiency in manufacturing I do not use it to calibrate θ directly.

[16] While all other parameters are taken from U.S. data, I match world rather than U.S. GDP so that the social planner internalizes global damages from CO_2 emissions.

Table 3: Base Year Model Calibration Summary

Parameter	Value	Source or calibration target
Internally calibrated parameters		
Aggregate productivity A_L	276.372	World GDP: \$66 trillion (2010 dollars)
Manufacturing capacity scale, each group	14,013.407	Manufacturing investment/GDP = 2.4%
Physical depreciation, durable / nondurable	4.030%, 4.746%	Implied I/K = 5.8%, 8.3%, Becker et al. (2021)
Manufacturing R&D cost shifter γ_m	3,720.890	Manufacturing fuel expenditure share = 5%
Dirty electricity fuel efficiency a_f	0.02694	Dirty electricity fuel share = 42%
Electricity CES weight ν	1.122×10^{10}	Electricity revenue/GDP = 1.86%
Clean electricity productivity A_g	0.27669	Clean electricity revenue share = 10%
Inverse scale of replacement cost shocks ζ_ϵ	5.0	Event study estimates
Externally imposed parameters		
Manufacturing weight in nonelectric composite κ	0.116	Approximate manufacturing value added/GDP
Durable capital CES weight β	0.286	Durable industry value added share, Becker et al. (2021)
Manufacturing capital exponent α	0.50	Manufacturing labor shares, Kehrig and Vincent (2021)
Electricity capital exponent φ	0.75	Giandrea and Sprague (2017)
Electricity/nonelectric substitution ϱ	0.10	Baseline assumption
Durable/nondurable capital substitution η	1.20	Oberfield and Raval (2021); Redding and Weinstein (2024)
Clean/fossil electricity output substitution ϑ	1.85	Papageorgiou et al. (2017)
Within sector machine variety substitution σ	4.0	Standard CES value
Knowledge depreciation δ_A	0.05	Benkard (2000); Hall (2007)
R&D curvature θ	0.35	Acemoglu et al. (2016); Bloom et al. (2013)
Electricity depreciation, fossil / clean	5%, 10%	Gilchrist et al. (2024)
Discount rate ρ	0.02	Imposed annual rate
Manufacturing fuel price proxy (2010 \$/MMBtu)	12.14	EIA SEDS TEICD, U.S. Energy Information Administration (2024)
Electricity sector fuel price proxy (2010 \$/MMBtu)	3.26	EIA SEDS PEEID, U.S. Energy Information Administration (2024)
Replacement opportunity rate	1.0/year	Normalization
Individual machine size $\bar{k}$	1.0	Normalization

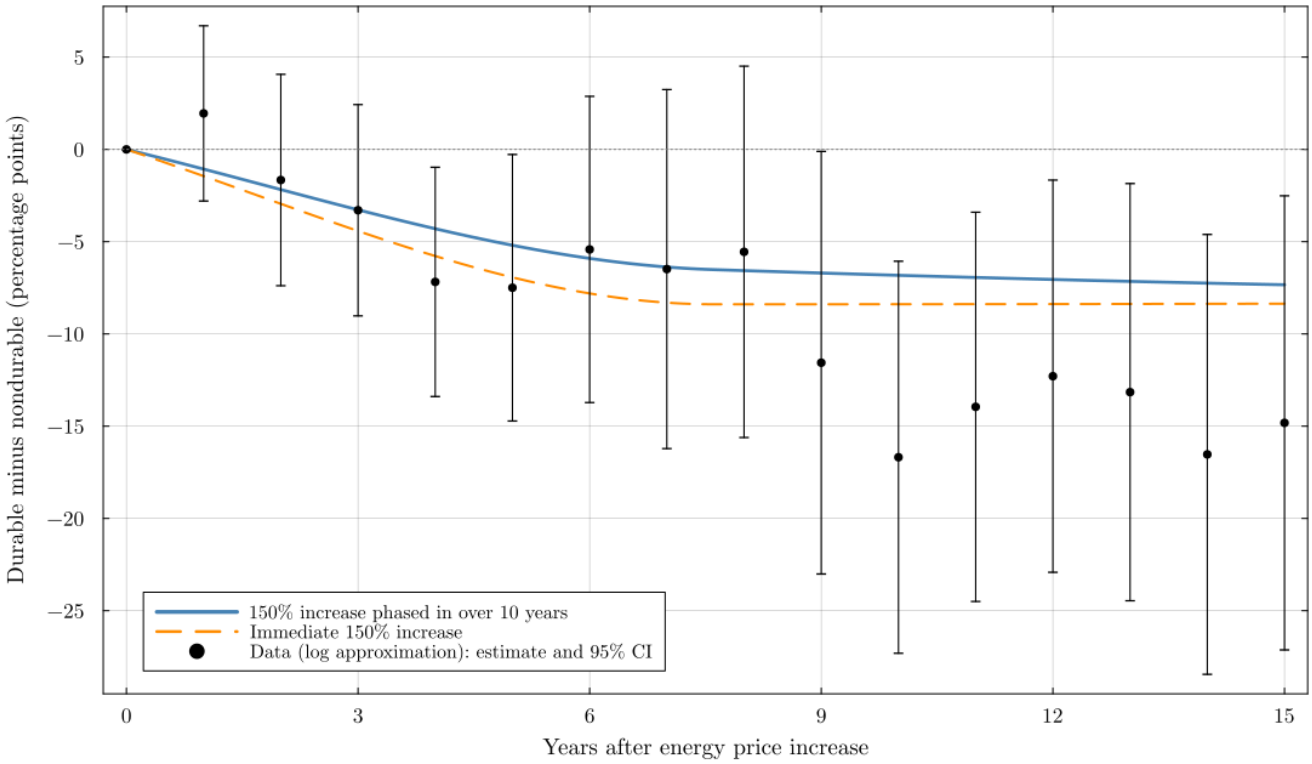


Figure 5: Differential Response of Energy Efficiency: Model versus Data

Notes. This figure compares the response of average energy efficiency in the model to the event study estimates in Figure 3(a). The solid line shows the response to an anticipated 150 percent increase in energy prices over ten years. The dashed line shows the response to an immediate increase of the same size. Average energy efficiency is weighted by the remaining productive capacity of machines. Empirical coefficients for 1974–1988 and their confidence intervals are scaled by the depreciation rate difference of 0.025, with 1973 normalized to zero. The y axis shows the difference in the efficiency response between durable and nondurable manufacturing, in percentage points. Empirical log changes are interpreted as approximate percentage changes. Vertical bars indicate 95 percent confidence intervals, constructed as in Figure 3(a).

3.3 Validation

The model matches new evidence on differential adjustment to energy prices presented in this paper as well as existing estimates. First, I shock the model with a permanent 150% energy price increase, mirroring the sustained increase of the early 1970s. I compare the model response with the rescaled event study estimates in Figure 3(a).[17] Figure 5 compares the differential adjustment paths between low and high depreciation industries in the model and the data. The figure shows the model response is broadly similar whether firms face an anticipated 10 year run up in energy prices, reaching 250% of the pre shock level, versus an immediate 150% increase.

The model's aggregate innovation response also fits external evidence. Popp (2002) estimates a long run elasticity of energy related patenting to energy prices of about 0.35. To compare this estimate with the model, I use an elasticity of patenting to R&D expenditure of 0.5, consistent with the estimates in Hall, Griliches, and Hausman (1986). In the model, permanent energy price increases of 30% and 150% generate long run elasticities of manufacturing research expenditure of 0.61 and 0.60, respectively. These

[17] I multiply event study coefficients in Figure 3(a) by the difference of 0.025 in depreciation rates between low and high depreciation sectors implied by the calibration.

imply patenting elasticities of approximately 0.30, close to Popp's estimate. The short run response depends more strongly on the size of the shock. Research expenditure rises by 13.1% on impact after the smaller price increase and by 22.4% after the larger increase, corresponding to elasticities of 0.47 and 0.22. Larger shocks induce sharper declines in equipment purchases, initially weakening the market size incentives for innovation.

Taking stock, the model assumes that energy and capital are perfect complements within each machine, but replacement allows energy efficiency to improve over time. The model reproduces the differential adjustment associated with capital durability and generates an aggregate innovation response broadly consistent with external evidence. I next use it to study how replacement and innovation shape the transition.

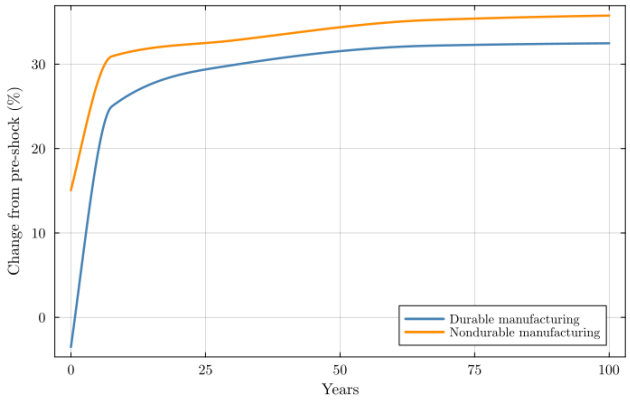
4 Quantitative Analysis

4.1 Energy Price Shocks and Model Mechanisms

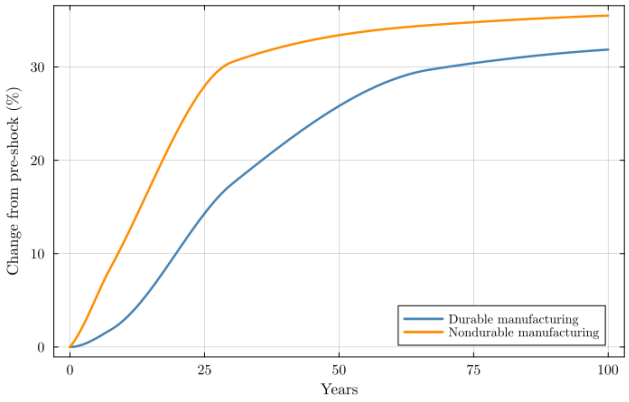
I start by analyzing the effects of a permanent, 150% increase in the price of energy. Energy price swings of this magnitude were observed in the 1970s (cf. Figure 2) or among European countries after the Ukraine war.[18] Figure 6 presents the transitional dynamics resulting from this shock. All variables are expressed relative to their pre shock steady state value. Results on the effects of energy price shocks abstract from climate economy feedback.[19]

[18] See data on industrial energy prices available from `https://www.gov.uk/government/statistical-data-sets/international-industrial-energy-prices`.

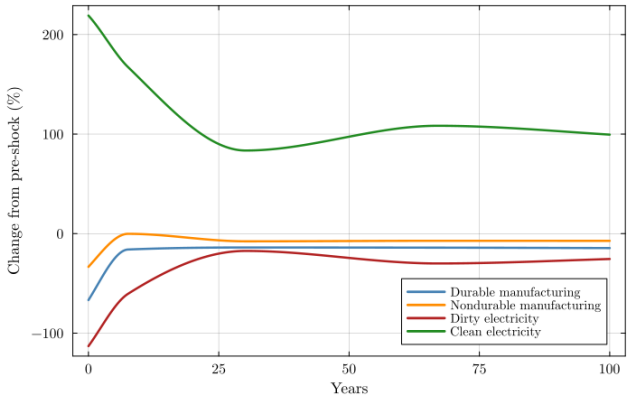
[19] This assumption matters for longer run outcomes because I calibrate the model to the world economy, so the feedback from low emissions to damages is nontrivial.



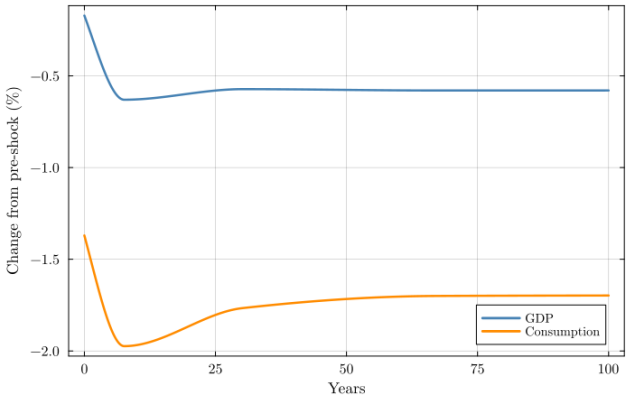
(a) Energy efficiency of new machines $A_e(t)$



(b) Average energy efficiency, $\bar{A}_e(t)$



(c) Investment



(d) Consumption and GDP

Figure 6: Transition Dynamics After a Permanent 150 Percent Increase in the Price of Energy

Notes. This figure presents simulated responses of sectoral and macroeconomic aggregates to a permanent 150 percent increase in the exogenous price of energy $p_e(t)$. The y axes measure percent changes relative to the value of each variable prior to the shock. The simulations do not account for feedback from emissions to climate damages.

Panels 6a and 6b show the evolution of $A_e(t)$, the energy efficiency embodied in new machines, and the average energy efficiency embodied in machines active in the market $\bar{A}_e(t) = \int_0^{\bar{k}} A(k)\mu(k,t)dk$. Energy efficiency of new machines increases relatively quickly for both manufacturing industries. These dynamics are in part driven by gradually rising R&D productivity via the law of motion for $\phi(t)$ in equation (22). Additionally, Figure 6c shows sharp initial declines in investment in fossil fuel using sectors. The initially low investment rates depress R&D incentives and even lead to a small decline in energy efficiency in durable manufacturing. Improvements in $A_e(t)$ translate into a drawn out increase in average efficiency $\bar{A}_e(t)$ (Figure 6b). The differential effect between durable and less durable manufacturing mirrors the event study results shown in Figure 5. The high depreciation sector has higher investment rates and exhibits faster convergence rates. The prolonged collapse in investment in durable manufacturing leads to a very slow transition of energy efficiency in that sector.

Turning to macroeconomic aggregates, Panel 6d shows that both consumption and output decline after the shock. The decline in consumption is more pronounced initially as higher energy prices force consumers to cut back immediately. As investment rises again, consumption further drops, and then recovers slightly as the response of technology partially undoes the effect on energy spending. Output is initially almost unaffected because capital is fixed in the short run and manufacturing remains fully utilized. Utilization falls only in fossil fuel electricity generation. The decline in investment then lowers capital and output over time. Effects on output and consumption are relatively moderate given that the fuel cost share in my calibrated model is only about 1.3%. The model omits energy spending on transport and buildings, which makes up a significant share of aggregate energy spending.

Market size effects and the sources of decarbonization. To quantify the role of innovation in driving decarbonization, Table 4 decomposes manufacturing fuel reductions into scale and technique effects. Scale measures lower fuel use from a smaller capital stock holding technology fixed at the pre shock level, while technique captures decreases in fuel use accounted for by innovation and adoption. Reductions in the capital stock account for about 40% of cumulative fuel reductions over the first 20 years, but the importance of this scale effect falls to about 20% in the long run, reflecting the time lag in adoption and innovation.

I examine the role of market size effects in two ways. First, I simply fix replacement rates at the pre shock level, shutting down the scale effect. This lowers cumulative fuel reductions over the first 20 years by about 15%, but substantially strengthens the response of innovation. Second, I introduce a research subsidy that compensates machine producers for changes in equipment demand, while leaving actual investment and replacement endogenous.[20] Figure D9 in Appendix D compares the sources of decarbonization when the market size effect is neutralized in this way versus the main model. Following

[20] In manufacturing sector i, the subsidy covers a fraction $s_i(t) = 1 - I_i(t)/I_i(0)$ of research costs, where $I_i(t)$ denotes equipment purchases in the subsidized equilibrium. With this subsidy the innovation condition becomes

$$A_{e,i}(t) = \left(\frac{\theta\phi_i(t)I_i(t)\tilde{p}_{e,i}(t)}{(1-s_i(t))\gamma}\right)^{\frac{\theta}{1+\theta}} = \left(\frac{\theta\phi_i(t)I_i(0)\tilde{p}_{e,i}(t)}{\gamma}\right)^{\frac{\theta}{1+\theta}}$$

effectively holding market size fixed at the pre shock level.

Table 4: Capital Replacement, Technology, and Manufacturing Fuel Savings

	First 20 years (cumulative)	Long run (steady state)
A. Manufacturing fuel savings (%)		
Main model	13.7	29.8
Fixed replacement hazard	11.6	27.4
B. Contributions to main model fuel savings (%)		
Smaller capital stock	40.4	18.7
Better embodied technology	59.6	81.3

Notes. This table reports manufacturing fuel reductions following a permanent 150 percent increase in energy prices. The first column measures cumulative reductions over the first 20 years, while the second reports the reduction in the new steady state. Both are measured relative to pre shock fuel use. Panel A compares the main model with a counterfactual that holds replacement hazards at their pre shock levels. Panel B decomposes fuel reductions in the main model into a smaller capital stock, holding energy efficiency fixed, and improvements in embodied technology. Contributions are summed across manufacturing sectors before calculating their shares.

the baseline 150% price increase, neutralizing the market size effect raises the cumulative fuel reduction attributable to better embodied technology by about one third over 20 years, increasing its contribution from 60% to 73%. These gains are partly offset by a smaller contraction in capital. Overall, the subsidy raises the cumulative 20 year reduction in manufacturing fuel use only modestly from 13.7% to 14.9% for the 150 percent price increase, but it plays a key role for the sources of rapid decarbonization. Figure D9 shows that market size effects on innovation grow with the energy price increase. The subsidy raises the share of fuel savings accounted for by technique by 6 percentage points after a 30% price increase and by 18 percentage points after a 200% increase. These findings illustrate that policy makers aiming to achieve rapid decarbonization through sharp policy interventions may increasingly rely on reductions in scale in the absence of policies counteracting the market size effect.

Energy price adjustment with high depreciation. Next, I show how capital durability affects the speed of the transition. I recalibrate the model so that implied depreciation in both manufacturing sectors equals 13.7%, retaining the remaining steady state calibration targets. An annual decay rate of 13.7% would imply a half life of about five years under exponential depreciation. I use this value as the target for implied depreciation in each manufacturing sector, a conservative benchmark relative to macro climate models that assume machines fully depreciate over 5 or 10 years (Acemoglu et al., 2012; Casey, 2023). Figure D10 in Appendix D compares manufacturing investment, average energy efficiency, and fuel use after the same permanent 150% increase in energy prices. Higher depreciation leads to faster improvements in energy efficiency and raises cumulative fuel savings over the first 20 years of the transition by about one third. In the long run, fuel use declines by almost the same amount in both economies. Differences in capital durability therefore matter primarily for how quickly the economy reaches its new steady state.

Table 5 separates the additional cumulative fuel savings into three contributions. First, higher depreciation accelerates adjustment towards a lower capital stock, as in the neoclassical growth model. I

Table 5: Additional Manufacturing Fuel Savings with Higher Depreciation

A. Extra cumulative fuel savings over 20 years	
High depreciation relative to main model	34.3%
B. Contributions to extra fuel savings	
Depreciation with fixed technology	28.2%
Faster adoption	61.6%
Endogenous innovation	10.2%

Notes. This table compares cumulative manufacturing fuel reductions over the first 20 years following a permanent 150 percent increase in energy prices. The counterfactual calibration sets implied depreciation to 13.7 percent in both manufacturing sectors. Panel A reports the percentage increase in fuel savings relative to the main model. Panel B decomposes these additional savings into higher depreciation holding technology fixed, faster adoption of the main model's technology, and the further effect of endogenous innovation. All counterfactuals are solved in general equilibrium.

measure this contribution by shutting down innovation, holding new machine efficiency fixed at its pre shock level. Second, higher depreciation allows firms to adopt improved technology more quickly. To isolate this effect, I impose the main model's relative technology growth on the high depreciation economy and measure the additional difference in savings beyond the fixed technology comparison. Third, higher replacement demand changes innovation incentives. I measure this contribution by allowing the high depreciation economy to follow its own technology path rather than the main model's relative technology growth. All counterfactuals are solved in general equilibrium, allowing investment, replacement decisions, and prices to adjust. Quantitatively, faster adoption accounts for about 62% of the additional cumulative fuel savings over the first 20 years. Differences in depreciation alone, holding technology fixed, contribute 28%, while greater innovation accounts for the remaining 10%. The larger savings therefore arise mainly because firms adopt better machines sooner.

Resale and larger spillovers. Two additional exercises highlight adoption behavior as a robust driver of these dynamics. First, I consider partial reversibility. If firms can resell their capital, they may be more easily induced to adopt new machines, alleviating lock in. To consider this possibility, I extend the model to include a fringe of firms that operates under free entry as potential customers of used capital. I assume resold capital depreciates at a higher rate, giving rise to empirically documented discounts in the price of old capital over its replacement cost (Kermani and Ma, 2023). Adding resale to the model changes the transmission of energy prices in several ways. First, the substitution effect to cleaner machines becomes stronger as firms purchasing new capital internalize the benefit of cleaner machines in the form of higher resale prices. Second, the market size effect becomes more negative as energy prices lower resale prices of old machines, adding a force that makes replacement less attractive after carbon prices go up. Finally, without changes to adoption and innovation in new machines, resale implies that old machines may remain in use, so that replacement may lead to even stronger increases in the scale of aggregate capital. Appendix B.3 provides details on this extension and its calibration. Figure D11 shows that the quantitative model with resale implies very similar decarbonization dynamics. Figure D12 shows similar responses of innovation, investment, and manufacturing fuel use. These results suggest that the opposing

effects on incentives to purchase new machines largely offset, while continued operation of old machines contributes little to aggregate fuel demand in this calibration.[21]

Second, I examine whether slow adoption remains important when innovation generates greater long run substitution away from fossil fuels. The baseline model's long run elasticity of substitution between capital and fossil fuels is below the commonly assumed unit elasticity that is often motivated as an outcome of innovation (Hassler et al., 2021). To assess the sensitivity of the transition dynamics to this difference, I modify the law of motion for $\phi(t)$ in equation (22) to double the partial equilibrium, steady state elasticity of new machine energy efficiency to energy prices. Appendix D describes the recalibration, and Figure D13 compares the resulting transition to the baseline. Larger spillovers generate substantially greater improvements in energy efficiency over time and a smaller long run decline in manufacturing investment. The initial investment contraction remains similar, however, and the additional fuel savings emerge gradually as firms replace existing machines. Over the first 25 years, larger spillovers reduce cumulative manufacturing fuel use by 17.7%, compared with 15.6% in the baseline, increasing cumulative savings by 13% (cf. Figure D11). Over the first 100 years, the corresponding reductions reach 33.6% and 25.1%. Thus, allowing for greater long run substitution does not eliminate the constraints imposed by slow replacement in the short and medium run.

4.2 Policy Analysis

4.2.1 Carbon Taxation

After illustrating the main mechanics of the model, I study their implications for climate policy. The economy features three distortions: a climate externality, intertemporal knowledge spillovers, and markups in the supply of machines. To isolate the effects of carbon pricing, I introduce a research subsidy and a subsidy to machine producers in both the business as usual economy and the economy with a carbon tax. The research subsidy reimburses 25% of research expenditure, corresponding to the steady state spillover correction (cf. Appendix D.3). Holding the subsidy constant, I do not solve for the fully optimal policy. A second subsidy to machine producers removes the markup, so the purchase price equals the resource cost of a new machine.

Both economies start from the calibrated steady state without these policies. The business as usual transition introduces the research subsidy and markup correction while allowing climate damages to evolve. The economy with a carbon tax features the same policies, together with a tax on emissions from manufacturing and electricity generation. All responses are measured relative to this common business as usual transition.

Carbon pricing. I set the carbon tax equal to the discounted marginal damage caused by an additional unit of emissions. Let $\widetilde{Y}(t) = D(S(t))Y(t)$ denote output after climate damages, where $D(S) =$

[21] Eisfeldt and Rampini (2007) report that average used equipment expenditure shares are below 10% for large firms.

$\exp[-g(S)]$, normalized to its initial value. With logarithmic utility, the social cost of carbon is

$$\Lambda(t) = \int_t^\infty e^{-\rho(s-t)} \frac{C(t)}{C(s)} g'(S(s)) \widetilde{Y}(s) G(s-t)\, ds, \tag{25}$$

where $G(s-t)$ measures the additional carbon remaining in the atmosphere at date s following a unit of emissions at date t.

Appendix B.4 derives the corresponding replacement condition and describes the subsidy implementation. Conditional on the technology path, correcting machine prices and charging firms the social cost of emissions aligns private replacement incentives with those of the planner. The research subsidy is held at its steady state rate throughout the transition rather than jointly optimized with the carbon tax. Since innovation spillovers are not fully internalized along the transition, the first best is not attained. The carbon tax is generally not constrained optimal since I do not solve for optimal second best carbon taxes.

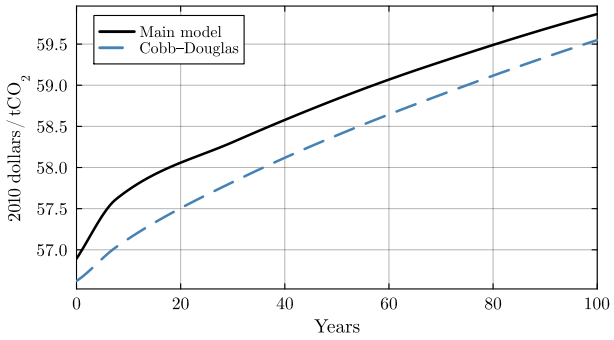
In the baseline specification, $g(S) = \psi(S - \bar{S})$, following Golosov et al. (2014). The marginal effect of carbon on log productivity is constant. Greater carbon accumulation therefore does not directly raise this slope, although the tax still depends on the saving rate via the consumption income ratio. Golosov et al. (2014) show that the transition dynamics in the savings rate are not very sensitive to different substitution elasticities in production, incomplete depreciation or intertemporal substitution. Thus, carbon taxes are largely a function of climate damage parameters and the rate of time preference.

Comparison with Cobb–Douglas. To quantify the carbon tax implications of replacement and innovation, I compare the main model with a neoclassical economy in which capital and fossil fuels enter production in Cobb–Douglas fashion. In this economy, firms can substitute away from fuel without replacing their machines. Energy saving technology is fixed and investment is fully reversible. I retain the sectoral structure and climate block and calibrate the benchmark to match the main model's initial capital stocks, fuel use, labor allocation, and factor income shares. Appendix B.5 provides the production functions and calibration details.

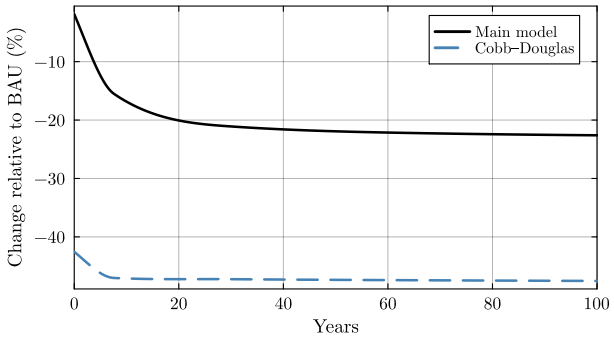
The Cobb–Douglas economy is evaluated relative to its own business as usual transition with evolving climate damages and the policy counterfactual adds the carbon tax under the same damage specification.

The top row of Figure 7 compares the resulting tax and emissions paths under baseline damages. Carbon taxes are similar across models, but emissions decline much more quickly in Cobb–Douglas. After 25 years, emissions from manufacturing and electricity generation are approximately 47% below business as usual, compared with 21% in the main model. Cobb–Douglas allows firms to reduce fuel use immediately. In the main model, a lack of immediate adoption and innovation makes emissions much more inelastic with respect to carbon taxes.

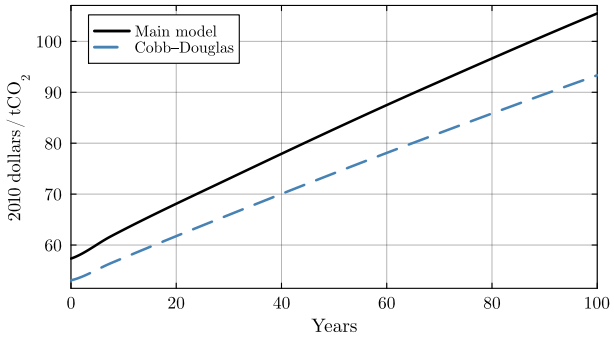
These differences also matter for welfare. Consumption initially rises in the main model as investment contracts, before the smaller capital stock lowers output. Over time, improvements in energy efficiency and lower climate damages raise consumption relative to business as usual. The consumption equivalent welfare gain is approximately 0.16%, compared with 0.38% in Cobb–Douglas. Appendix Figure D14



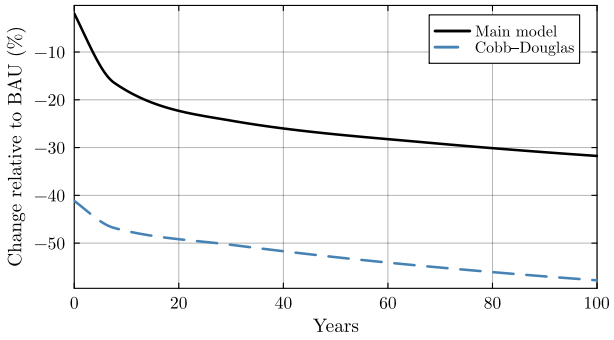
(a) Baseline damages: carbon tax



(b) Baseline damages: emissions



(c) Convex damages: carbon tax



(d) Convex damages: emissions

Figure 7: Carbon Taxes and Emissions under Baseline and Convex Damages

Notes. This figure compares carbon taxes and emissions in the main model and the Cobb–Douglas economy. The upper panels use the baseline damage function, while the lower panels allow marginal damages to increase with carbon accumulation. Emissions include manufacturing and fossil fuel electricity generation and are expressed as percentage changes relative to each economy's business as usual path. In the main model, both the tax and business as usual paths introduce a research subsidy to internalize steady state innovation spillovers and a subsidy that removes the markup in machine production. Carbon taxes are measured in 2010 dollars per tonne of CO_2.

Table 6: Carbon Taxes, Emissions, and Welfare

	Main model	Cobb–Douglas	Main model Matching tax
Carbon tax at year 25 (2010 dollars/tCO_2)	58.2	57.7	251.3
Emissions reduction at year 25 (%)	20.7	47.2	47.2
Welfare gain (% of consumption)	+0.16	+0.38	-0.42

Notes. This table compares carbon taxes, emissions, and welfare under the baseline damage function. The first two columns use the carbon tax defined in equation (25). The third imposes a constant tax in the main model that matches the Cobb–Douglas economy's emissions reduction after 25 years. Emissions reductions and welfare gains are measured relative to each economy's business as usual path. Welfare is expressed as a consumption equivalent gain over 500 years and includes the benefits of lower climate damages.

reports the responses of consumption, GDP, investment, and average energy efficiency.

Next, I ask how high a constant carbon tax in the main model must be to achieve the Cobb–Douglas economy's emissions reduction after 25 years. The required tax is about 4.4 times the Cobb–Douglas tax at that date. Table 6 shows that achieving this reduction entails a welfare loss in the main model, despite the benefits of lower climate damages. Since climate benefits of the policy are roughly equal across models by construction, this comparison illustrates the output cost of rapid decarbonization when capital replacement is central. Appendix Figure D15 compares the emissions, manufacturing output, and consumption paths under the three policies. These results are consistent with Casey (2023) who shows that short run frictions raise the taxes required to meet near term emissions targets.

Increasing marginal damages. Under the Golosov et al. (2014) specification of climate damages higher emissions along the transition do not affect the social cost of carbon. This leaves the carbon tax largely insensitive to replacement innovation feedback. To examine how relaxing this assumption can affect optimal taxes, I follow Campiglio et al. (2022) and compute carbon taxes using the following specification for damages

$$D(S) = \exp\left[-\psi_2(S - \bar{S})^2\right]. \tag{26}$$

The marginal effect on log productivity is now $g'(S) = 2\psi_2(S - \bar{S})$, which rises as carbon accumulates. I use the same damage parameter in both economies and compare each policy path to its corresponding business as usual transition. Appendix B.4.1 provides the calibration and numerical details.

The bottom row of Figure 7 shows that the main model now implies a higher carbon tax. Slow replacement delays emissions reductions, raising atmospheric carbon and hence marginal damages. After 25 years, the tax is approximately 11% higher than in Cobb–Douglas, while the emissions reduction remains substantially smaller. Capital replacement therefore determines how effective carbon taxes are and also leads to a carbon tax premium when marginal damages are increasing.

4.2.2 Replacement Subsidies

Governments often subsidize durable manufacturing investment, for example through bonus depreciation (Curtis, Garrett, Ohrn, Roberts, and Suarez-Serrato, 2023; Zwick and Mahon, 2017). Such policies can

stimulate innovation by expanding the market for new machines, but also encourage the accumulation of fossil fuel consuming capital. I first consider uniform replacement subsidies and then study whether targeting electric machinery can generate larger fuel reductions.

I calibrate a permanent subsidy to manufacturing equipment purchases to approximately match the fiscal scale of bonus depreciation, measured relative to initial GDP over ten years. This yields a subsidy of 4.1%.[22] The innovation and scale effects largely offset, leaving manufacturing fuel use approximately unchanged in Figure 8(b).[23]

4.2.3 Electrification

Electric machinery offers an alternative to relying on improvements in fuel efficiency that have yet to be developed. I study the effectiveness of carbon taxes and replacement subsidies when firms can electrify production. While adoption still requires capital replacement, I assume an electrified alternative exists, i.e. I abstract from innovation incentives. To study this margin, I extend the firm investment problem to allow firms to switch between fuel types when they receive a replacement opportunity. Let $V_F(k,t)$ denote the value of a fossil fuel machine and $V_E(k,t)$ the value of an electric machine. Suppressing the manufacturing sector index, the Bellman equations become

$$r(t)V_E(k,t) = \pi_E(k,t) - \delta k \partial_k V_E(k,t) + \partial_t V_E(k,t) + \lambda\big(\mathcal{V}_E(k,t) - V_E(k,t)\big), \tag{27}$$

$$r(t)V_F(k,t) = \pi_F(k,t) - \delta k \partial_k V_F(k,t) + \partial_t V_F(k,t) + \lambda\big(\mathcal{V}_F(k,t) - V_F(k,t)\big). \tag{28}$$

A replacement opportunity now allows firms to purchase a machine using the same fuel, switch fuel types, or continue operating the current machine. Letting $P_f(t)$ and $P_e(t)$ denote the purchase prices of fossil fuel and electric machines, the expected values of the replacement opportunity are

$$\begin{aligned}\mathcal{V}_F(k) = \mathbb{E}_\varepsilon\big[\max\{&V_F(\bar{k}) - (1-s_f)\bar{k}P_f + \varepsilon^F_{\text{rep}},\\ &V_E(\bar{k}) - (1-s_e)\bar{k}P_e + \varepsilon^F_{\text{sw}}, \quad V_F(k) + \varepsilon^F_{\text{keep}}\}\big],\end{aligned} \tag{29}$$

$$\begin{aligned}\mathcal{V}_E(k) = \mathbb{E}_\varepsilon\big[\max\{&V_E(\bar{k}) - (1-s_e)\bar{k}P_e + \varepsilon^E_{\text{rep}},\\ &V_F(\bar{k}) - (1-s_f)\bar{k}P_f + \varepsilon^E_{\text{sw}}, \quad V_E(k) + \varepsilon^E_{\text{keep}}\}\big].\end{aligned} \tag{30}$$

Here π_F and π_E denote operating profits. The subsidies s_f and s_e reimburse a share of the purchase price of fossil fuel and electric machines. Both are zero in the carbon tax experiment, while the targeted subsidy sets $s_f = 0$ and $s_e > 0$. Electric machines are supplied competitively and have fixed electricity requirements. I start by assuming electricity used to power these machines is imported at a constant

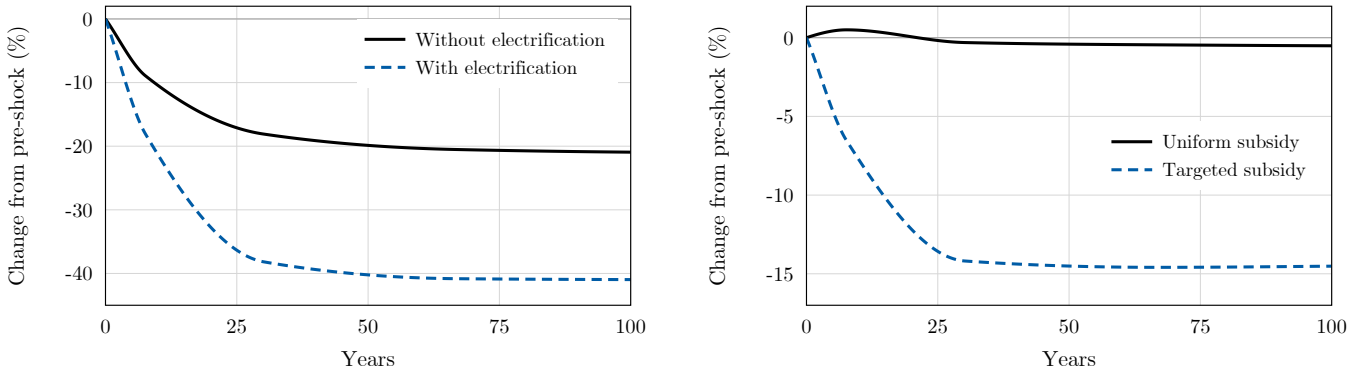
[22] Curtis et al. (2023) cite a projected fiscal cost of $285 billion over 2019–2028. Dividing by 2019 US GDP gives a ten year budget of about 1.3% of initial GDP. I allocate this budget to manufacturing. Appendix E.4 explains the calibration.

[23] While scale effects largely offset the benefits of faster adoption under uniform replacement subsidies, directly encouraging innovation can substantially reduce fuel use. A research subsidy set at the steady state spillover correction of 25% reduces manufacturing fuel use by 7.3% after 25 years (Appendix Figure D16). I focus on replacement subsidies as the role of research subsidies for green transition has been studied extensively (e.g., Acemoglu et al., 2012).

price and relax this assumption below. I calibrate electric machine costs to a 13% electricity expenditure share in manufacturing in line with US data. Appendix E provides operating profits, the evolution of the machine distributions and calibration details.

Figure 8(a) compares the response to a permanent carbon tax of approximately \$40/tCO_2. Allowing for electrification increases cumulative manufacturing fuel reductions over the first 25 years from 11% to 23%. Access to electric machinery also limits the initial investment contraction.[24] The electrification response is broadly consistent with the reduced form findings of Kaartinen and Prane (2024). Their event study shows an increase of about ten percentage points in electricity's share of energy use over five years following the Swedish tax increases. In the model, the share of manufacturing capital using electricity rises by eight percentage points over the same horizon.

Electrification also creates scope for targeted replacement subsidies. In Figure 8(b), I subsidize purchases of electric machines, holding total subsidy expenditure over the first ten years equal to the uniform subsidy in the main model.[25] This requires a 11.3% subsidy and reduces manufacturing fuel use by about 14% after 25 years. Subsidizing an existing clean alternative therefore generates substantially greater fuel reductions at the same fiscal cost.



(a) Carbon tax, with and without electrification

(b) Replacement subsidies at equal fiscal cost

Figure 8: Manufacturing Fuel Use under Carbon Taxes and Replacement Subsidies

Notes. This figure shows the response of manufacturing fossil fuel use to carbon taxes and replacement subsidies. Panel (a) compares the main model and the electrification extension under a permanent carbon tax of approximately \$41 per tonne of CO_2, in 2010 dollars. The tax applies to manufacturing and electricity generation. Panel (b) compares a uniform replacement subsidy in the main model with a subsidy to electric machines in the extension, holding total subsidy expenditure over the first ten years constant. Electricity used by these machines is imported at a constant price in the electrification economy. The y axes show percentage changes relative to each economy's pre shock steady state. The simulations abstract from climate damages.

Finally, Table 7 compares carbon taxes and electrification subsidies calibrated to reduce manufacturing

[24] Manufacturing investment falls by 26% on impact without electrification, compared with 4% when electric machinery is available. Investment in fossil fuel machines falls while purchases of electric machines increase. This additional margin is consistent with Brown et al. (2026), who find that higher carbon prices raise capital investment and the share devoted to abatement among high emission firms. Figure E1 reports investment and adoption.

[25] These effects are quantitatively robust to comparing uniform and electrification subsidies within the model that allows for both electrification and innovation.

fuel use by 10%, 20%, and 30% after 25 years. Both policies are evaluated against the same business as usual economy with evolving climate damages and no research subsidy or markup correction. Subsidies encourage electric investment, whereas carbon taxes also price continued fossil fuel use. The two policies therefore generate different investment and consumption paths even when they meet the same fuel target. Overall, subsidies yield lower welfare, with the gap reported in consumption equivalent percentage points. Carbon taxes effectively target the underlying externality, while electrification subsidies partially directly distort investment. Thus, subsidies can act as an alternative, but at the cost of only indirectly addressing the climate externality.

Table 7: Carbon Taxes and Electrification Subsidies at Matched Fuel Reductions

Fuel reduction at year 25	Carbon tax (2010 \$/tCO$_2$)	Electrification subsidy (%)	Welfare gap (tax minus subsidy, pp)
10%	11.1	9.1	0.086
20%	23.4	14.6	0.168
30%	35.0	18.8	0.229

Notes. This table reports the carbon taxes and subsidies to electric machines required to reduce manufacturing fossil fuel use by 10, 20, and 30 percent after 25 years. Both policies are evaluated in the electrification economy relative to the same business as usual path, without research subsidies or subsidies to correct machine markups. Carbon taxes apply to manufacturing fuel use. Electricity is imported at a constant price. The welfare gap measures how much larger the consumption equivalent welfare gain is under carbon taxation than under the subsidy, in percentage points. Welfare includes climate damages over 500 years but excludes emissions from producing imported electricity.

The aggregate emissions benefits depend on how the additional electricity is produced. Appendix E.5 allows electricity to be supplied domestically. The targeted subsidy produces almost the same manufacturing fuel reduction, indicating that higher electricity prices crowd out little of the fuel savings. However, additional power sector emissions offset about 39% of direct emissions savings after 25 years. These fuel savings are a function of lower emissions intensity in electricity generation which is partially clean. Manufacturing fuel use still falls substantially more than without electrification. Overall, these results strongly support the idea that electrification presents a promising alternative to energy saving technical change, especially as electricity generation increasingly relies on clean energy (Arkolakis and Walsh, 2023).

5 Conclusion

The transition towards less fossil fuel intensive production requires replacing slowly depreciating capital with lower emissions vintages. I show empirically that capital durability shapes these transition dynamics. Industries with less durable capital experience larger improvements in energy efficiency following energy price shocks. These improvements are accompanied by increases in green innovation, consistent with feedback between adoption and innovation. To study how this feedback affects the green transition, I calibrate a multisector IAM with vintage capital and endogenous improvements in energy augmenting technology. The complementarity between capital and energy combined with the slow adoption of new vintages makes energy demand highly inelastic in the short run. Carbon taxes increase incentives to improve energy efficiency, but the associated decline in investment limits demand for new machines and dampens the innovation response. These findings illustrate that slow adoption and its feedback to innovation limit the power of technology to lower emissions in the short run.

To spur adoption, policy makers frequently provide manufacturing firms with generous investment subsidies. My model captures an important benefit of such subsidies: by raising investment, these subsidies incentivize innovation. Model simulations show that these efficiency improvements are largely offset by scale effects, leaving manufacturing fuel use approximately unchanged. While these findings caution against the use of uniform subsidies, I also consider subsidies targeted at electrifying manufacturing firms. Electrification allows firms to adopt an existing alternative rather than relying on further improvements in fossil fuel efficiency. Targeting subsidies generates substantially larger reductions in manufacturing fuel use at the same fiscal cost. The overall emissions benefits depend on whether the additional electricity is supplied by sufficiently clean generation.

Overall, this paper is part of an emerging literature emphasizing the role of capital adjustment costs for climate dynamics (Bertolotti et al., 2026; Dietz et al., 2021). An open question is whether additional spillovers arise when firms switch towards cleaner vintages. My model considers spillovers to innovation, but other forms of spillovers may render firm replacement decisions socially inefficient even after the distortions considered here are corrected. An example would be lower switching costs if many firms adopt simultaneously. The existence of such spillovers would further justify policies that coordinate and accelerate the transition to clean technology.

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A Data and Robustness Checks

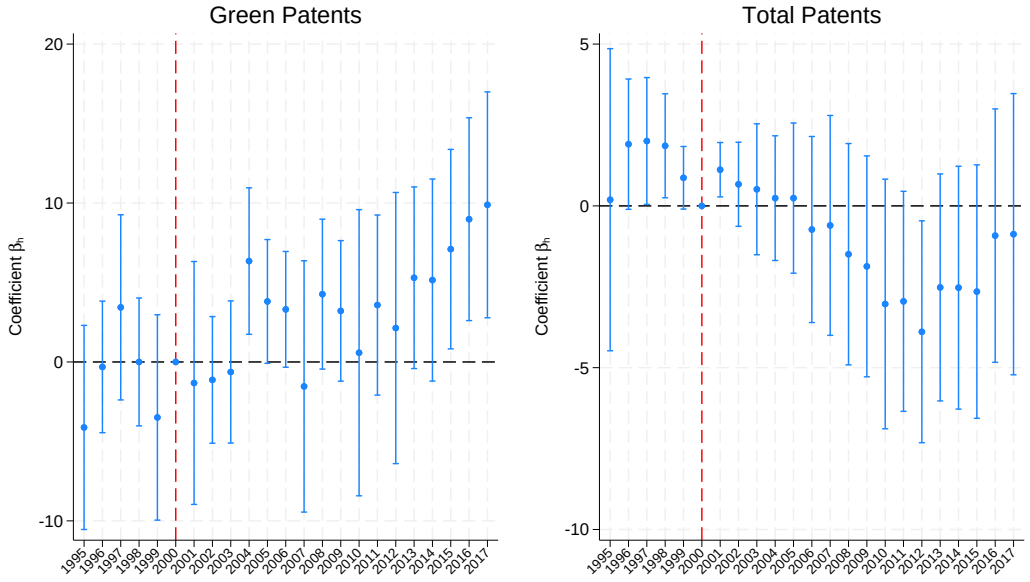
A.1 Methods.

Depreciation Rates. National and industry level economic accounts construct asset j capital stocks based on the formula $K_{j,t} = \sum_{j}^{S_j} \phi_{j,i} I_{j,t-1}$ where $i \in \{1, S_j\}$ denotes how many years ago the investment took place with S_j being the maximum possible service life (Gilbert and Mohr, 1996). The parameters ϕ summarize the underlying age efficiency function: How much does an investment from year i contribute to the current capital stock j in year t? To calibrate this extended perpetual inventory method, the Federal Reserve Bank combines BLS data on mean service lives of individual assets with a probabilistic model of discard and decay of capital goods. This calibration goes back to pioneering work by Hulten and Wykoff (1981) that used prices of used equipment to determine rates of decay.

The perpetual inventory method in equation (1) simplifies this model by assuming a constant rate of decay as capital ages. Depreciation rates obtained from inverting (1) thus partially capture changes in the relative shares of vintages. A boom in investment may lower depreciation since the Fed model assumes that new assets depreciate less than older assets. Since investment is highly cyclical, I average the depreciation rates across the post 1990 period to smooth out such effects.

A.2 Robustness Checks

Figure A1: Second energy price shock - (Log) Green and Total Patents



Note: This figure plots coefficients $\{\beta_t\}_{t=-5}^{15}$ around the increase in the real price of energy paid by industrial users starting in 2000. The dependent variables are (log) green patents in the left panel and (log) total patents in the right panel. Both panels use the continuous depreciation rate measure as the exposure variable. Standard errors are clustered at the four digit NAICS level.

Table A1: First Stage

	(1)
Fixed Weight log(p)	0.461***
	(5.97)
Industry F.E.	yes
Year F.E.	yes
Clusters	81
F Statistic	35.6
Observations	2268

t statistics in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Note: This Table shows the first stage regressions results from regressing industry level energy prices on the fixed weight energy price index used as an instrumental variable in Table 2.

Table A2: OLS results

	E/VA			Share Green Patents		
	(1)	(2)	(3)	(4)	(5)	(6)
Lagged Outcome	0.859***	0.855***	0.879***	0.577***	0.561***	0.537***
	(32.69)	(31.27)	(26.15)	(6.20)	(6.07)	(5.51)
log(p)	-0.0188	-0.0201	0.160*	-0.127	-0.131	-0.391**
	(-0.36)	(-0.38)	(1.97)	(-1.41)	(-1.40)	(-2.27)
log(p) x delta		-2.063**	-2.223***		10.41***	9.143***
		(-2.13)	(-2.84)		(3.31)	(2.82)
Industry F.E.	yes	yes	yes	yes	yes	yes
Year F.E.	yes	yes	yes	yes	yes	yes
Controls	no	no	yes	no	no	yes
Clusters	81	81	81	81	81	81
Observations	2187	2187	2106	2187	2187	2106

t statistics in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Note: This Table shows OLS estimates of the differential effects of industry level energy prices on the (log) ratio of real energy use divided by real value added (Columns (1)-(3)) and the (log) share of green patents (Columns (4)-(6)). Columns (4) and (6) additionally control for the interaction of baseline energy intensity and capital intensity with industry level energy prices. Standard errors are clustered at the four digit NAICS level.

Table A3: OLS results: Biadic Patents

	E/VA			Share Green Patents		
	(1)	(2)	(3)	(4)	(5)	(6)
Lagged Outcome	0.859***	0.855***	0.879***	0.529***	0.516***	0.495***
	(32.63)	(31.28)	(26.13)	(5.90)	(5.85)	(5.35)
log(p)	-0.0160	-0.0172	0.162**	-0.127	-0.130	-0.404**
	(-0.30)	(-0.32)	(1.99)	(-1.24)	(-1.21)	(-2.08)
log(p) x delta		-2.033**	-2.173***		10.73***	9.282***
		(-2.09)	(-2.77)		(3.20)	(2.68)
Industry F.E.	yes	yes	yes	yes	yes	yes
Year F.E.	yes	yes	yes	yes	yes	yes
Controls	no	no	yes	no	no	yes
Clusters	80	80	80	80	80	80
Observations	2160	2160	2080	2160	2160	2080

t statistics in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Note: This Table shows OLS estimates of the differential effects of industry level energy prices on the (log) ratio of real energy use divided by real value added (Columns (1)-(3)) and the (log) share of biadic green patents (Columns (4)-(6)). Columns (4) and (6) additionally control for the interaction of baseline energy intensity and capital intensity with industry level energy prices. Effects in columns (1)-(3) differ from A2 because the sample is restricted to industries with nonzero biadic patents in all years. Standard errors are clustered at the four digit NAICS level.

Table A4: OLS results: Alternative duration measure

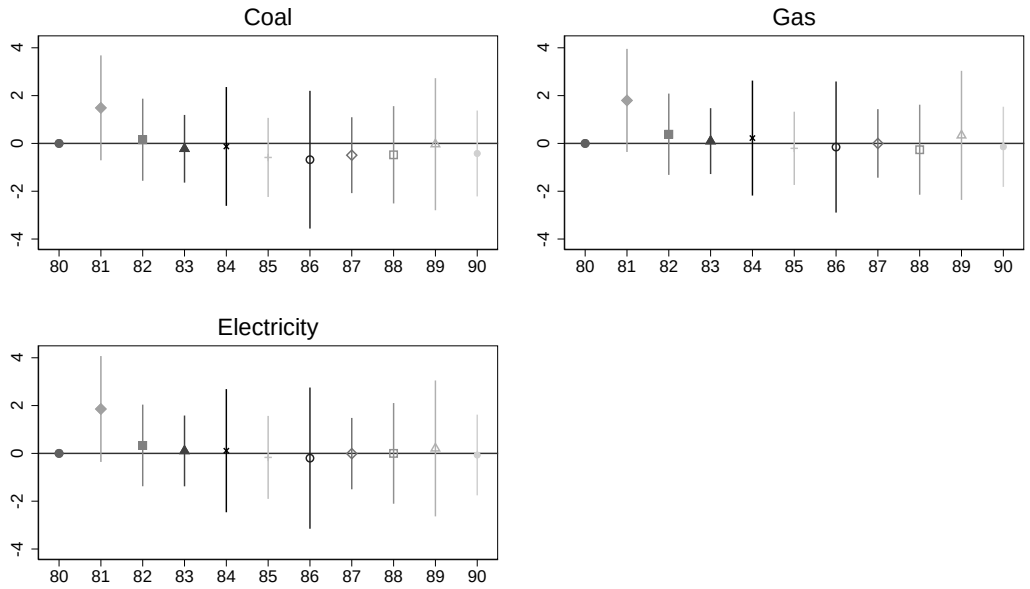
	E/VA			Share Green Patents		
	(1)	(2)	(3)	(4)	(5)	(6)
Lagged Outcome	0.859***	0.861***	0.887***	0.577***	0.565***	0.540***
	(32.69)	(32.71)	(27.76)	(6.20)	(5.67)	(5.08)
log(p)	-0.0188	-0.0269	0.179**	-0.127	-0.136	-0.386**
	(-0.36)	(-0.50)	(2.02)	(-1.41)	(-1.45)	(-2.17)
log(p) x T		0.00230	0.00930**		-0.0315**	-0.0288*
		(0.52)	(2.17)		(-2.39)	(-1.96)
Industry F.E.	yes	yes	yes	yes	yes	yes
Year F.E.	yes	yes	yes	yes	yes	yes
Controls	no	no	yes	no	no	yes
Clusters	81	72	72	81	72	72
Observations	2187	1944	1872	2187	1944	1872

t statistics in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

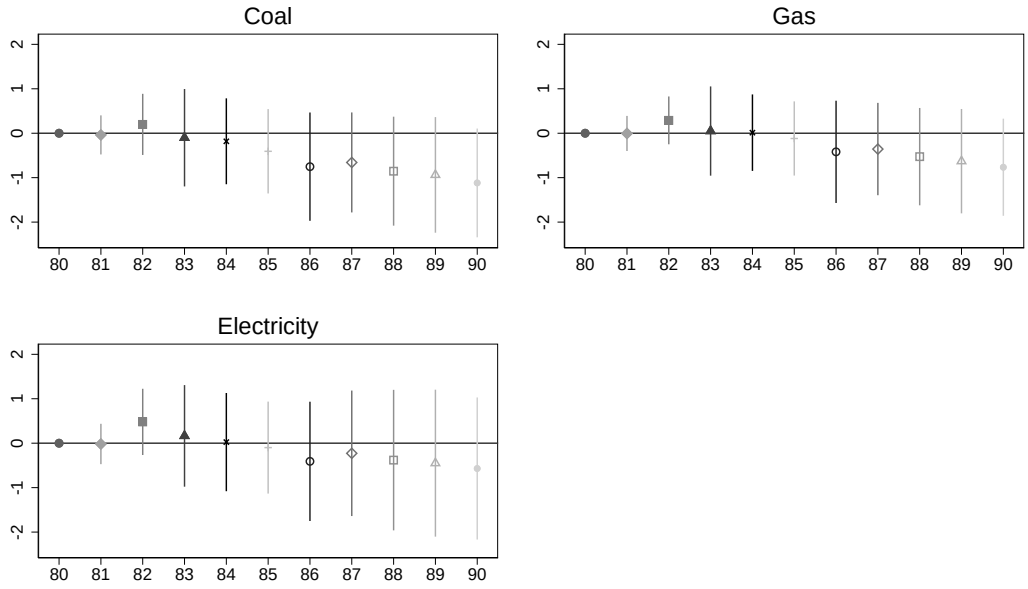
Note: This table shows OLS estimates of the differential effects by durability of industry level energy prices on the (log) ratio of real energy use divided by real value added (Columns (1)-(3)) and the (log) share of green patents (Columns (4)-(6)). Columns (4) and (6) additionally control for the interaction of baseline energy intensity and capital intensity with industry level energy prices. Durability is defined as the weighted average of industry asset level service lives reported by the BLS. Standard errors are clustered at the four digit NAICS level.

Figure A2: Pretrends of fuel shares: patents



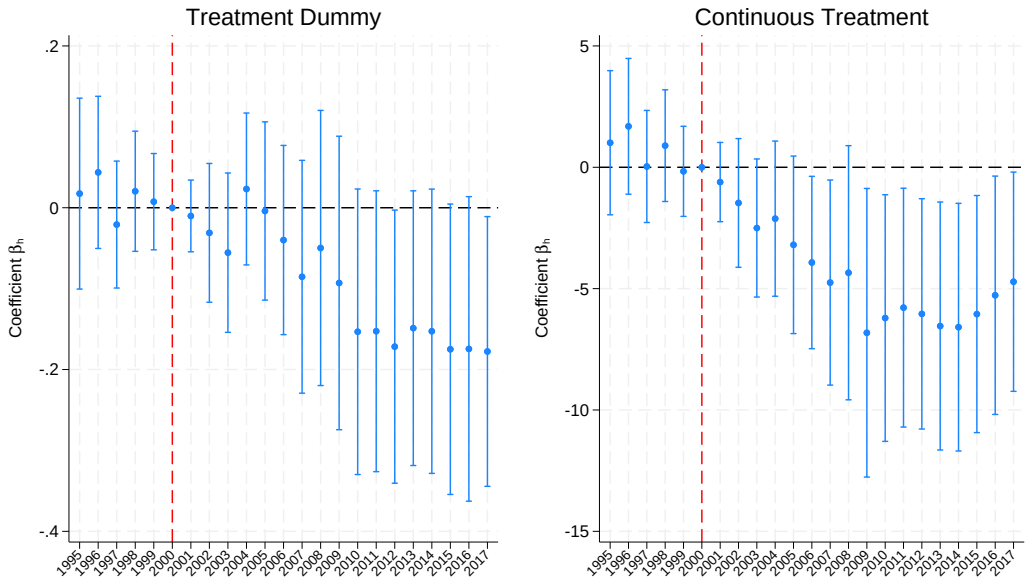
Note: This Figure shows differential effects of industry level fuel shares on the (log) ratio of green to total patents prior to 1990.

Figure A3: Pretrends of fuel shares: energy intensity



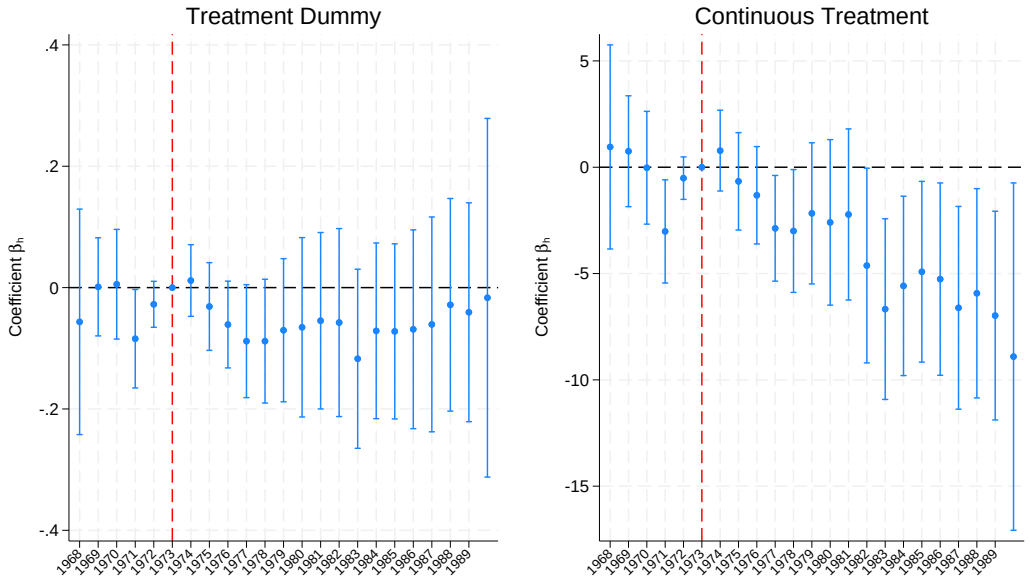
Note: This Figure shows differential effects of industry level fuel shares on the (log) ratio of energy to value added prior to 1990.

Figure A4: Discrete versus continuous exposure: Second energy price shock - energy intensity



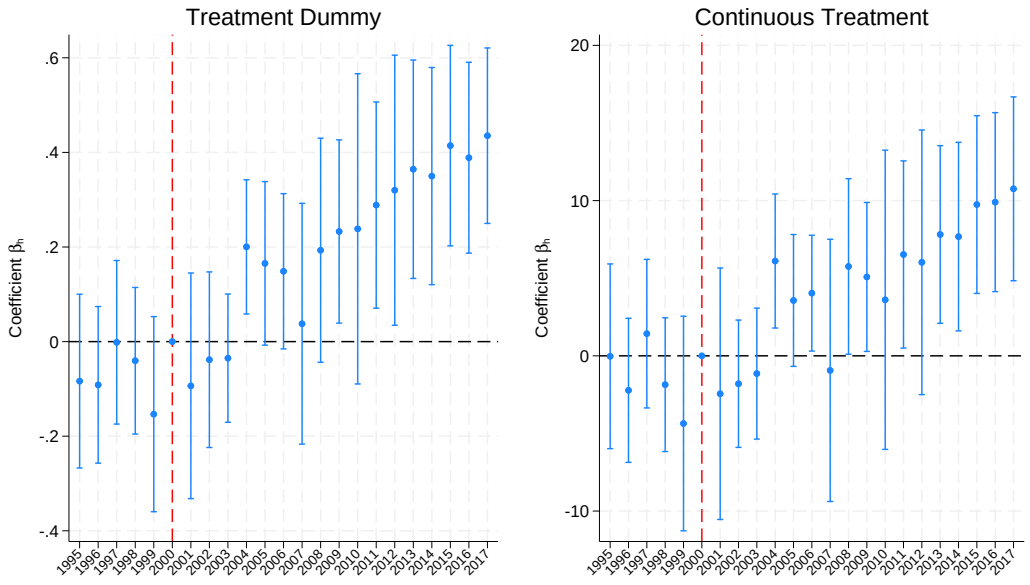
Note: This Figure plots coefficients $\{\beta_t\}_{t=-5}^{15}$ around the increase in the real price of energy paid by industrial users starting in 2000. The dependent variable is the (log) ratio of real energy use divided by real value added. The right hand side panel shows effects using the continuous depreciation rate variable as the measure of exposure. The left hand side panel uses a dummy equal to one for industries with depreciation rate above the 75th percentile of the depreciation rate distribution across industries. Standard errors are clustered at the four digit NAICS level.

Figure A5: Discrete versus continuous exposure: First energy price shock - energy intensity



Note: This Figure plots coefficients $\{\beta_t\}_{t=-5}^{15}$ around the increase in the real price of energy paid by industrial users starting in 1973. The dependent variable is the (log) ratio of real energy use divided by real value added. The right hand side shows effects using the continuous depreciation rate variable as the measure of exposure. The left hand side uses a dummy equal to one for industries with depreciation rate above the 75th percentile of the depreciation rate distribution across industries. Standard errors are clustered at the four digit NAICS level.

Figure A6: Discrete versus continuous exposure: Second energy price shock - share of green patents



Note: This Figure plots coefficients $\{\beta_t\}_{t=-5}^{15}$ around the increase in the real price of energy paid by industrial users starting in 2000. The dependent variable is the (log) ratio of green to total patents. The right hand side shows effects using the continuous depreciation rate variable as the measure of exposure. The left hand side uses a dummy equal to one for industries with depreciation rate above the 75th percentile of the depreciation rate distribution across industries. Standard errors are clustered at the four digit NAICS level.

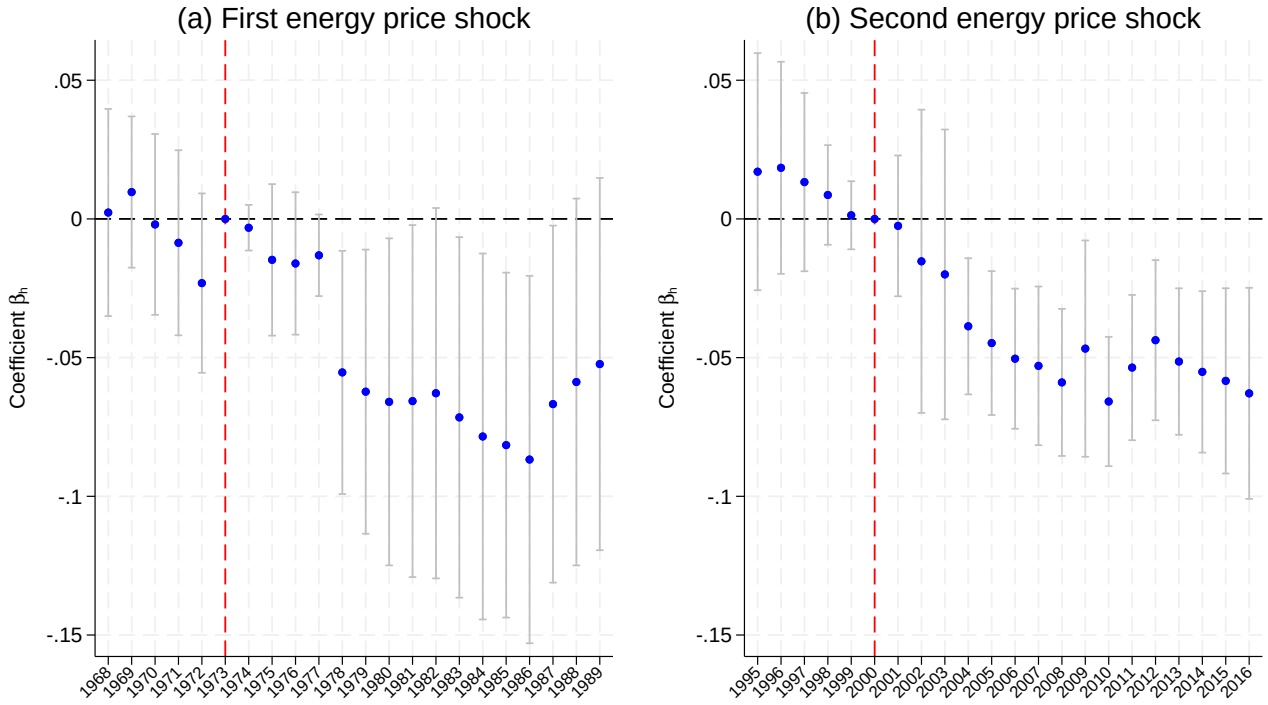
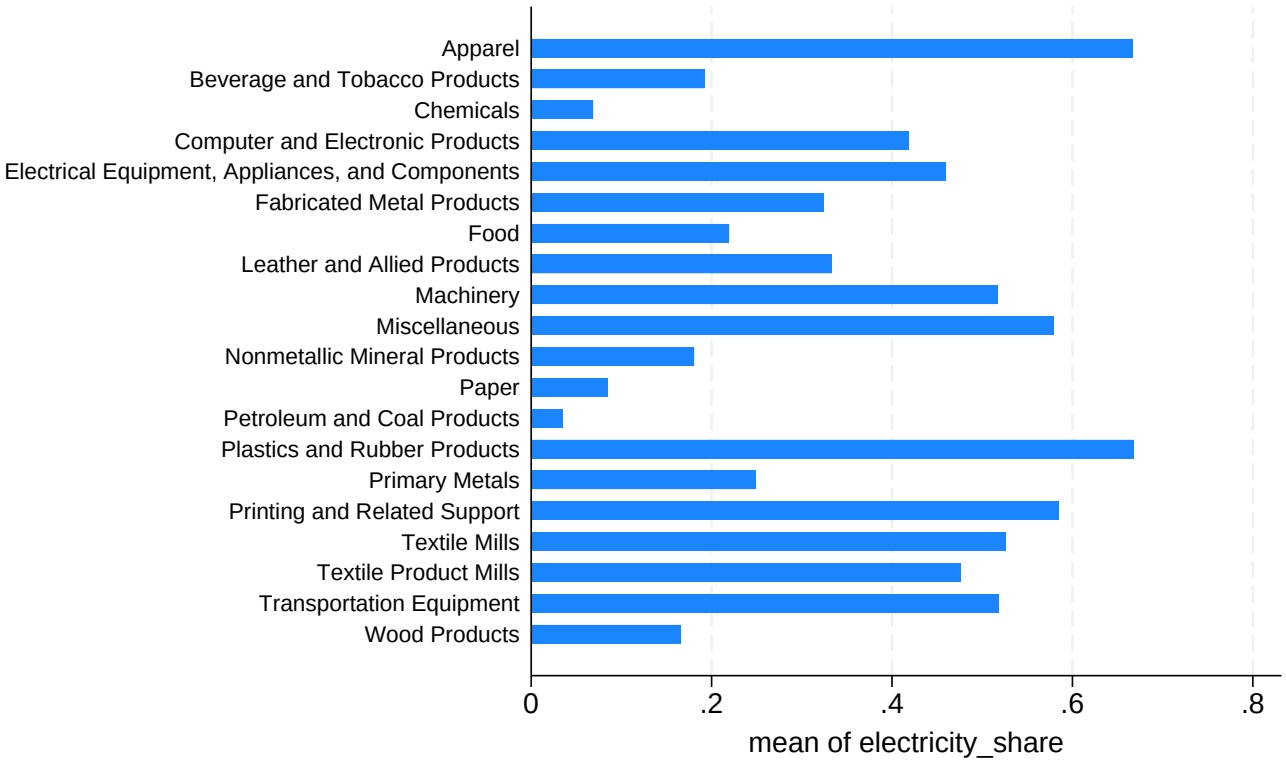


Figure A7: Equipment purchases and capital durability

Note: This Figure plots coefficients $\{\beta_t\}$ around the two increases in the real price of energy paid by industrial users. The exposure variable is equipment service life, measured in years using the same BLS service life schedules underlying the alternative industry durability measure. The dependent variable is (log) real equipment purchases by manufacturing firms, aggregated within each of 23 BEA equipment categories. Panel (a) shows effects around the first energy price increase starting in 1973. Panel (b) shows effects for the second energy price increase starting in 2000. Both regressions include equipment and year fixed effects, with coefficients normalized to zero in 1973 and 2000, respectively. Vertical bars indicate 95% confidence intervals. Standard errors are clustered at the equipment category level.

Figure A8: Sectoral Share of Electricity in Total Energy Consumption



Note: This figure plots the distribution of the share of electricity in total energy consumption across 3 digit NAICS manufacturing sectors in the U.S. Data is from the Manufacturing Energy Consumption Survey.

B Additional Derivations

B.1 Derivations of the HJB and KFE

I suppress the sector index. Capital depreciates according to $\dot{k} = -\delta k$, and firms receive replacement opportunities at rate λ. Replacement returns capital to $\bar{k}$ at cost $P(t)\bar{k} - \epsilon$.

Derivation of the HJB. The expected value at a replacement opportunity is

$$\mathcal{V}(k,t) = \mathbb{E}_\epsilon \max\{V(\bar{k},t) - P(t)\bar{k} + \epsilon, V(k,t)\}.$$

Over a small interval dt, the Bellman equation can be written as

$$V(k,t) = \pi(k,t)dt + (1 - r(t)dt)\Big[(1-\lambda dt)V(k(1-\delta dt), t+dt) + \lambda dt \mathcal{V}(k(1-\delta dt), t+dt)\Big] + o(dt).$$

Using $V(k(1-\delta dt), t+dt) = V(k,t) + (V_t - \delta k V_k)dt + o(dt)$, subtracting $V(k,t)$ and dividing by dt gives

$$r(t)V(k,t) = \pi(k,t) + V_t(k,t) - \delta k V_k(k,t) + \lambda[\mathcal{V}(k,t) - V(k,t)]. \tag{31}$$

The last term is the expected gain from a replacement opportunity, allowing the firm to continue operating its existing machine when replacement is not attractive.

Derivation of the distribution. Let $\mu(k,t)$ denote the density of firms over capital, with total mass N. Define the cumulative mass $F(k,t) = \int_0^k \mu(x,t)\,dx$. For an interior point $0 < k < \bar{k}$, firms below k at $t + dt$ are those with capital below $k/(1-\delta dt)$ at t that do not replace. Hence

$$F(k, t+dt) = \int_0^{k/(1-\delta dt)} [1 - \lambda P^R(x,t)dt]\mu(x,t)\,dx + o(dt),$$

where $P^R(k,t)$ is the probability of replacing conditional on an opportunity. Subtracting $F(k,t)$, dividing by dt and taking the limit gives

$$F_t(k,t) = \delta k \mu(k,t) - \lambda \int_0^k P^R(x,t)\mu(x,t)\,dx.$$

Differentiating with respect to k yields

$$\mu_t(k,t) = \partial_k[\delta k \mu(k,t)] - \lambda P^R(k,t)\mu(k,t). \tag{32}$$

Firms that replace reenter at $\bar{k}$, giving the boundary condition

$$\delta \bar{k}\mu(\bar{k},t) = M(t), \qquad M(t) = \lambda \int_0^{\bar{k}} P^R(k,t)\mu(k,t)\,dk \tag{33}$$

that equates the flow of firms replacing their capital with the inflow of capital at $\bar{k}$. Together with zero flux at $k = 0$, this condition preserves the mass of firms. Gross equipment purchases are $I(t) = \bar{k}M(t)$.

B.2 Derivations of Machine Demand and Supply

Demand for machines. A new machine consists of a bundle of varieties $k(\nu, t)$ whose CES aggregate equals $\bar{k}$. As in the main text, firms purchase machines anticipating full utilization until replacement. The energy inclusive price of variety ν is

$$\widetilde{p}(\nu, t) = p(\nu, t) + \frac{\widetilde{p}_e(t)}{A_e(\nu, t)}, \qquad \widetilde{p}_e(t) = \int_t^\infty e^{-H(t,\tau)} p_e(\tau)\, d\tau,$$

where

$$H(t, \tau) = \delta(\tau - t) + \int_t^\tau \left[r(u) + \lambda P^R(\bar{k}e^{-\delta(u-t)}, u) \right] du.$$

The discount factor accounts for physical depreciation, interest rates and the probability that the machine is replaced. A change in one variety has no effect on the replacement of the entire bundle. Each producer therefore takes H, aggregate purchases and the aggregate price index as given.

Let $\xi(t)$ be the multiplier on the constraint that the CES aggregate of the new machine equals $\bar{k}$. The first order condition for $k(\nu, t)$ is

$$\left[\int_t^\infty e^{-H(t,\tau)} P^K(\tau)\, d\tau - \xi(t) \right] \bar{k}^{1/\sigma} k(\nu, t)^{-1/\sigma} = \widetilde{p}(\nu, t).$$

Substituting this condition into the CES constraint implies

$$\int_t^\infty e^{-H(t,\tau)} P^K(\tau)\, d\tau - \xi(t) = \widetilde{P}(t), \qquad \widetilde{P}(t) = \left[\int_0^1 \widetilde{p}(\nu, t)^{1-\sigma} d\nu \right]^{1/(1-\sigma)}.$$

Demand for each variety is therefore

$$k(\nu, t) = \bar{k} \left[\frac{\widetilde{p}(\nu, t)}{\widetilde{P}(t)} \right]^{-\sigma}. \tag{34}$$

Pricing. The producer sells $M(t)k(\nu, t)$ units at margin $p(\nu, t) - c$. Since research expenditure does not depend on the purchase price, the pricing condition is

$$1 - \sigma \frac{p(\nu, t) - c}{\widetilde{p}(\nu, t)} = 0.$$

Using the definition of the energy inclusive price gives

$$p(\nu, t) = \frac{\sigma}{\sigma - 1} c + \frac{1}{\sigma - 1} \frac{\widetilde{p}_e(t)}{A_e(\nu, t)}. \tag{35}$$

Innovation. Research expenditure is $R(\nu, t) = \gamma A_e(\nu, t)^{1/\theta} / \phi(t)$. At the optimal price, the envelope

condition implies that the marginal benefit of energy efficiency is

$$M(t)[p(\nu,t)-c]\frac{\partial k(\nu,t)}{\partial A_e(\nu,t)} = M(t)k(\nu,t)\frac{\widetilde{p}_e(t)}{A_e(\nu,t)^2}.$$

Equating this benefit to marginal research cost gives

$$M(t)k(\nu,t)\frac{\widetilde{p}_e(t)}{A_e(\nu,t)^2} = \frac{\gamma}{\theta\phi(t)}A_e(\nu,t)^{1/\theta-1}. \tag{36}$$

In a symmetric equilibrium $k(\nu,t)=\bar{k}$. Rearranging (36) gives

$$A_e(t) = \left[\frac{\theta\phi(t)M(t)\bar{k}\widetilde{p}_e(t)}{\gamma}\right]^{\theta/(1+\theta)}. \tag{37}$$

Higher replacement demand raises the return to the fixed research expenditure required to improve the energy efficiency of new machines. The symmetric price index is

$$\widetilde{P}(t) = \frac{\sigma}{\sigma-1}\left[c+\frac{\widetilde{p}_e(t)}{A_e(t)}\right].$$

At a positive stationary point of the producer's problem, the strict second order condition is

$$\sigma\frac{\widetilde{p}_e(t)/A_e(t)}{c+\widetilde{p}_e(t)/A_e(t)} < 1+\frac{1}{\theta}.$$

This condition follows by differentiating the producer's profit after substituting the optimal price, holding aggregate purchases, the price index and research productivity fixed.

Electricity aggregation. The parameter ϑ is the elasticity of substitution between clean and fossil fuel electricity outputs. To derive the corresponding capital aggregate, denote the capital and labor inputs in the two technologies by K_f, K_g, L_f and L_g, suppressing time. Here K_f and K_g are the operated capital inputs $K_e(f,t)$ and $K_e(g,t)$. Electricity output is

$$X_e = \left[(K_f^{\varphi}(A_L L_f)^{1-\varphi})^{\rho_e} + A_g(K_g^{\varphi}(A_L L_g)^{1-\varphi})^{\rho_e}\right]^{1/\rho_e}, \qquad \rho_e = \frac{\vartheta-1}{\vartheta}.$$

Given total electricity labor $L_e = L_f + L_g$, its allocation maximizes X_e. Define $d_e = 1-(1-\varphi)\rho_e$ and $\eta_e = \varphi\rho_e/d_e$. Equalizing the marginal products of labor implies

$$\frac{L_g}{L_f} = A_g^{1/d_e}\left(\frac{K_g}{K_f}\right)^{\eta_e}.$$

Substitution into the production function yields

$$X_e = K_e^{\varphi}(A_L L_e)^{1-\varphi}, \qquad K_e = \left(K_f^{\eta_e} + A_g^{1/d_e}K_g^{\eta_e}\right)^{1/\eta_e}. \tag{38}$$

The elasticity between the two capital inputs is $1/(1-\eta_e) = 1+\varphi(\vartheta-1)$. With $\varphi = 0.75$ and $\vartheta = 1.85$, it equals 1.6375. Labor reallocation therefore matters for mapping substitution between electricity outputs into substitution between capital inputs.

B.3 Resale of Used Capital

I extend the model to include a competitive fringe of firms that stand ready to purchase used machines. Fringe firms face the same output and energy prices as incumbent capital users. They operate acquired machines when profitable, but do not purchase new machines or solve a replacement problem. Machines retain their embodied energy efficiency after resale, while their remaining capital depreciates at rate $\delta_F > \delta$, capturing the fact that machines may be subject to a higher rate of failure after transfer from the original user.

Resale values and replacement. $q(k,t)$ denotes the resale value of a machine held by an incumbent with capital k. The machine retains its embodied energy efficiency $A(k,t)$ upon resale. Since there is an infinite mass of potential entrants, free entry implies that $q(k,t)$ must equal the discounted value of operating the machine. In steady state, when operation is profitable, this gives

$$q(k) = \frac{k}{r+\delta_F}\left(P^K - \frac{p_e}{A_e}\right). \tag{39}$$

Resale values are proportional to remaining capital and increasing in embodied energy efficiency. Higher depreciation in the fringe lowers the value of used machines, in line with empirically documented price discounts in the secondary market.

The incumbent's replacement problem is now modified to account for the value of the sale of existing capital

$$r(t)V(k,t) = \pi(k,t) - \delta k V_k(k,t) + V_t(k,t) + \lambda\big(\mathcal{V}(k,t) - V(k,t)\big), \tag{40}$$

where

$$\mathcal{V}(k,t) = \mathbb{E}_\epsilon\left[\max\left\{V(\bar{k},t) - P(t)\bar{k} + q(k,t) + \epsilon,\, V(k,t)\right\}\right]. \tag{41}$$

The firm replaces whenever

$$V(\bar{k},t) - P(t)\bar{k} + q(k,t) + \epsilon > V(k,t). \tag{42}$$

The replacement probability therefore becomes

$$P^R(k,t) = \mathbb{P}\left(V(\bar{k},t) - P(t)\bar{k} + q(k,t) - V(k,t) > -\epsilon\right). \tag{43}$$

The incumbent distribution and definition of gross investment are not affected as purchases by fringe firms transfer existing machines between owners.

The fringe distribution. Let $\mu^F(k,t)$ denote the density of fringe firms over the remaining capital of their machines. Its law of motion is

$$\partial_t \mu^F(k,t) = \partial_k[\delta_F k \mu^F(k,t)] + \lambda P^R(k,t)\mu(k,t)\mathbf{1}_{\{q(k,t)>0\}}. \tag{44}$$

The first term reflects depreciation of machines already in the fringe. The second adds machines sold by incumbents when they replace. Machines with zero resale value are scrapped. Importantly, aggregate fuel use is now defined through these two distributions, i.e. it accounts for fuel use of incumbents and the fringe.

Demand for new machines and innovation. The presence of the fringe affects the demand for new machines. A more energy efficient machine is relatively more valuable because the firm benefits not only through lower fuel costs while the machine is in operation, but also from the higher resale price of a more efficient machine.

To describe the steady state innovation condition, define

$$T = \int_0^\infty \exp\left[-(r+\delta)a - \int_0^a \lambda P^R(\bar{k}e^{-\delta s})\, ds\right] da. \tag{45}$$

This is the discounted operating horizon with the original owner, accounting for physical depreciation and replacement. Including subsequent operation in the fringe gives

$$T^F = T + \frac{1-(r+\delta)T}{r+\delta_F}. \tag{46}$$

The numerator of the second term is the expected discounted remaining capital at replacement, per unit of initial capital. Dividing by $r+\delta_F$ gives its subsequent operating horizon in the fringe. The replacement probabilities determining T are those of the resale economy.

The effective lifetime fuel price becomes

$$\tilde{p}_e^F = p_e T^F.$$

Defining T in this way, we can write demand again as

$$k(\nu) = \bar{k}\left(\frac{\tilde{p}(\nu)}{\tilde{P}}\right)^{-\sigma}, \qquad \tilde{p}(\nu) = p(\nu) + \frac{\tilde{p}_e^F}{A_e(\nu)}, \tag{47}$$

where

$$\tilde{P} = \left(\int_0^1 \tilde{p}(\nu)^{1-\sigma}\, d\nu\right)^{1/(1-\sigma)}.$$

Industry demand for variety ν is $Mk(\nu)$. In a symmetric equilibrium, the innovation condition is

$$A_e = \left(\frac{\theta \phi M \bar{k} \, \tilde{p}_e^F}{\gamma} \right)^{\theta/(1+\theta)}. \tag{48}$$

Using the steady state knowledge stock $\phi = A_e/\delta_A$, this can be written as

$$A_e = \left(\frac{\theta M \bar{k} \, p_e T^F}{\gamma \delta_A} \right)^{\theta}. \tag{49}$$

As in the baseline model, innovation depends on the market for new machines and the value of energy savings. Resale changes both replacement demand and the horizon over which efficiency improvements remain valuable.

Transmission of energy prices. Resale changes energy price transmission through three channels. First, more efficient machines command higher resale prices, allowing buyers to capture fuel savings beyond their own replacement cycle. This extends the horizon over which energy efficiency is valuable. Second, higher energy prices reduce the resale value of existing machines, holding other prices and technology fixed. This raises the net cost of replacement and can weaken innovation through lower demand for new equipment. Third, machines sold to the fringe remain available for production, so replacement need not eliminate the fuel use of old vintages.

Calibration. I calibrate the fringe depreciation rate to a recovery rate of 30%, motivated by Kermani and Ma (2023).[26] The corresponding model moment is

$$\frac{q(k)}{Pk} = \frac{P^K - p_e/A_e}{(r + \delta_F)P} = 0.30. \tag{50}$$

A machine originally purchased for $P\bar{k}$ has depreciated book value Pk. The denominator therefore accounts for the capital already depreciated, rather than measuring recovery relative to the original purchase cost.

I impose the same recovery target in both manufacturing sectors and recalibrate the economy to retain the baseline targets, including investment to capital ratios of 5.8% and 8.3%. The resulting fringe depreciation rates are 23.3% in durable manufacturing and 52.1% in nondurable manufacturing.

B.4 Derivations of the Planner Problem

To characterize the efficient replacement policy, I follow the distributional approach of Nuño and Moll (2018). To economize on notation, I state the problem for a single fuel using sector and suppress the remaining production and labor allocation decisions. The derivations extend to each manufacturing sector separately.

[26] They report a 30% liquidation value for industrial PPE. This provides a benchmark for the value of capital in alternative use.

The social planner problem. The planner chooses consumption, replacement, utilization, and the energy efficiency of new machines to maximize

$$\max_{\{C,h,u,A_e\}} \int_0^\infty e^{-\rho t} \log C(t)\, dt.$$

Here $h(k,t,\epsilon) \in \{0,1\}$ indicates replacement following an opportunity with shock ϵ, and $f(\epsilon)$ denotes the density of that shock. Consumption satisfies $C(t) > 0$, and utilization satisfies $u(k,t) \in [0,1]$. The planner is subject to the resource constraint

$$\widetilde{Y}(t) + B_\epsilon(t) = C(t) + cI(t) + R(t) + p_e(t)E(t), \tag{51}$$

where $\widetilde{Y}(t) = D(S(t))Y(t)$ is output after climate damages and $p_e(t)$ is the resource price of fuel. Fuel use is

$$E(t) = \int_0^{\bar{k}} \frac{u(k,t)k}{A(k,t)} \mu(k,t)\, dk,$$

where $A(k,t)$ is the energy efficiency embodied in a machine with capital k at time t. Gross investment is $I(t) = \bar{k}M(t)$ and research expenditure is $R(t) = \gamma A_e(t)^{1/\theta}/\phi(t)$. Replacement shocks change the resource cost of installing a new machine, with aggregate contribution

$$B_\epsilon(t) = \lambda \int_0^{\bar{k}} \mu(k,t) \int h(k,t,\epsilon)\epsilon f(\epsilon)\, d\epsilon\, dk.$$

Taxes and subsidies are transfers and do not enter this constraint.

Replacement probability is $P^R(k,t) = \int h(k,t,\epsilon) f(\epsilon)\, d\epsilon$, and the mass of firms replacing is $M(t) = \lambda \int_0^{\bar{k}} P^R(k,t)\mu(k,t)\, dk$. The distribution of capital evolves according to

$$\partial_t \mu(k,t) = \partial_k[\delta k \mu(k,t)] - \lambda P^R(k,t)\mu(k,t), \qquad 0 < k < \bar{k},$$

with boundary condition $\delta \bar{k} \mu(\bar{k},t) = M(t)$. Research productivity evolves according to $\dot{\phi}(t) = A_e(t) - \delta_A \phi(t)$. Atmospheric carbon is $S(t) = S_P(t) + S_D(t)$, with

$$\begin{aligned} \dot{S}_P(t) &= \varphi_L \mathcal{E}(t), \\ \dot{S}_D(t) &= -\varphi_S S_D(t) + (1-\varphi_L)\varphi_0 \mathcal{E}(t). \end{aligned}$$

Here $\mathcal{E}(t) = \chi E(t) + \mathcal{E}^{\text{exo}}(t)$ denotes total carbon emissions, χ converts fuel use into carbon emissions, and $\mathcal{E}^{\text{exo}}(t)$ denotes emissions outside the modeled sector.

Derivation of the social marginal value function $j(k,t)$. Let $\lambda^C(t)$ denote the current value multiplier on the resource constraint, $j(k,t)$ the shadow value associated with the distribution of capital, and $\lambda^\phi(t)$ the shadow value of research productivity. The consumption first order condition gives $\lambda^C(t) = 1/C(t)$.

The terms in the Lagrangian associated with the law of motion for μ are

$$\int_0^\infty e^{-\rho t} \int_0^{\bar{k}} j(k,t)\big\{\partial_k[\delta k \mu(k,t)] - \lambda P^R(k,t)\mu(k,t) - \partial_t \mu(k,t)\big\}\, dk\, dt.$$

Integrating by parts in k gives the boundary term $j(\bar{k},t)\delta\bar{k}\mu(\bar{k},t) = j(\bar{k},t)M(t)$. Integrating by parts in time, imposing the transversality condition on j and μ, and omitting the fixed initial term gives

$$\int_0^\infty e^{-\rho t} \int_0^{\bar{k}} \mu(k,t)\Big\{\partial_t j(k,t) - \rho j(k,t) - \delta k \partial_k j(k,t) + \lambda P^R(k,t)[j(\bar{k},t) - j(k,t)]\Big\}\, dk\, dt.$$

Combining these terms with investment costs and replacement shocks, all terms involving $h(k,t,\epsilon)$ are proportional to

$$\lambda\mu(k,t)h(k,t,\epsilon)[j(\bar{k},t) - j(k,t) - \lambda^C(t)(c\bar{k} - \epsilon)].$$

The planner therefore replaces a machine whenever

$$j(\bar{k},t) - j(k,t) > \lambda^C(t)(c\bar{k} - \epsilon). \tag{52}$$

The replacement decision compares the social value of installing a new machine with the value of retaining the old machine, accounting for the resource cost of replacement.

The expected replacement value is $\mathcal{J}(k,t) = \mathbb{E}_\epsilon \max\{j(\bar{k},t) - \lambda^C(t)(c\bar{k} - \epsilon), j(k,t)\}$. Substituting the optimal replacement decision and differentiating with respect to the distribution gives

$$\rho j(k,t) = \lambda^C(t)u(k,t)k\left[P^K(t) - \frac{p_e(t) + \chi\Lambda(t)}{A(k,t)}\right] + \partial_t j(k,t) - \delta k \partial_k j(k,t) + \lambda[\mathcal{J}(k,t) - j(k,t)], \tag{53}$$

where $P^K(t) = \partial\widetilde{Y}(t)/\partial K^u(t)$ is the marginal product of utilized capital after damages.

The social cost of carbon, $\Lambda(t)$, values the effect of an additional unit of emissions on future output. Writing $D(S) = \exp[-g(S)]$, it equals

$$\Lambda(t) = \int_t^\infty e^{-\rho(\tau - t)} \frac{\lambda^C(\tau)}{\lambda^C(t)} g'(S(\tau))\widetilde{Y}(\tau) \times [\varphi_L + (1 - \varphi_L)\varphi_0 e^{-\varphi_S(\tau - t)}] d\tau.$$

This expression applies to both damage specifications considered in the paper.

Innovation. Improving the efficiency of machines purchased at time t lowers fuel use for as long as those machines remain in operation. At date τ, a surviving machine has capital $k(\tau;t) = \bar{k}e^{-\delta(\tau - t)}$. The

surviving mass of the cohort is related to the density of firms by

$$M(t)\exp\left[-\int_t^\tau \lambda P^R(k(s;t),s)\,ds\right] = \delta k(\tau;t)\mu(k(\tau;t),\tau).$$

A marginal improvement in this cohort's technology changes subsequent fuel use by

$$\frac{\delta E(\tau)}{\delta A_e(t)} = -\frac{I(t)e^{-\delta(\tau-t)}}{A_e(t)^2}\exp\left[-\int_t^\tau \lambda P^R(k(s;t),s)\,ds\right] u(k(\tau;t),\tau).$$

Using the consumption Euler equation (4) to discount these fuel savings, the direct marginal benefit of innovation in goods is $I(t)\widetilde{p}_e(t)/A_e(t)^2$, where

$$\widetilde{p}_e(t) = \int_t^\infty e^{-H(t,\tau)}[p_e(\tau)+\chi\Lambda(\tau)]u(k(\tau;t),\tau)\,d\tau,$$
$$H(t,\tau) = \delta(\tau-t) + \int_t^\tau [r(s)+\lambda P^R(k(s;t),s)]\,ds.$$

Under the maintained assumption that new machines are fully utilized until replacement, this is the lifetime energy price from Section 3, including the carbon tax.

Innovation also raises research productivity, reducing the cost of future research. The terms in the Lagrangian involving $\phi(t)$ are

$$\mathcal{L}_\phi = \int_0^\infty e^{-\rho t}\left[-\lambda^C(t)\gamma\frac{A_e(t)^{1/\theta}}{\phi(t)} + \lambda^\phi(t)(A_e(t)-\delta_A\phi(t)-\dot{\phi}(t))\right]dt.$$

Integrating the term involving $\dot{\phi}(t)$ by parts gives

$$\begin{aligned}-\int_0^\infty e^{-\rho t}\lambda^\phi(t)\dot{\phi}(t)\,dt = &-\left[e^{-\rho t}\lambda^\phi(t)\phi(t)\right]_0^\infty \\ &+\int_0^\infty e^{-\rho t}[\dot{\lambda}^\phi(t)-\rho\lambda^\phi(t)]\phi(t)\,dt.\end{aligned}$$

The initial research stock is given, and the terminal boundary term vanishes under the transversality condition. Differentiating with respect to $\phi(t)$ gives

$$\dot{\lambda}^\phi(t) = (\rho+\delta_A)\lambda^\phi(t) - \lambda^C(t)\frac{R(t)}{\phi(t)}, \qquad \lambda^\phi(t) = \int_t^\infty e^{-(\rho+\delta_A)(\tau-t)}\lambda^C(\tau)\frac{R(\tau)}{\phi(\tau)}\,d\tau.$$

An additional unit of research productivity lowers research costs by R/ϕ. Its social value is the discounted value of these future savings. Differentiating the Lagrangian with respect to $A_e(t)$ and dividing by $\lambda^C(t)$ gives the planner's innovation condition

$$\frac{R(t)}{\theta A_e(t)} = \frac{I(t)\widetilde{p}_e(t)}{A_e(t)^2} + \frac{\lambda^\phi(t)}{\lambda^C(t)}. \tag{54}$$

The marginal research cost equals the value of lower fuel use for the current cohort plus the value of reducing future research costs.

Decentralization of replacement. Suppose machine users pay the resource cost c per unit of new capital and face the fuel price $p_e(t)+\chi\Lambda(t)$. Substituting $j(k,t) = \lambda^C(t)V(k,t)$ and $r(t) = \rho - \dot{\lambda}^C(t)/\lambda^C(t)$ into the social value equation gives the private replacement HJB

$$r(t)V(k,t) = u(k,t)k\left[P^K(t) - \frac{p_e(t)+\chi\Lambda(t)}{A(k,t)}\right] + \partial_t V(k,t) - \delta k \partial_k V(k,t) + \lambda[\mathcal{V}(k,t) - V(k,t)],$$

where $\mathcal{V}(k,t) = \mathbb{E}_\epsilon \max\{V(\bar{k},t) - c\bar{k} + \epsilon, V(k,t)\}$. The private and social replacement rules thus coincide. Conditional on the technology path, replacement is efficient when machine prices equal resource costs and firms pay the social cost of their emissions.

Machine producer subsidy. To correct machine markups, let $b(t)$ denote a subsidy per unit of capital sold. An individual producer takes this subsidy as given and solves

$$\max_{p(\nu,t),A_e(\nu,t)} M(t)k(\nu,t)[p(\nu,t) + b(t) - c] - (1 - s_R(t))\gamma\frac{A_e(\nu,t)^{1/\theta}}{\phi(t)},$$

subject to the demand for varieties derived in Section 3. The price first order condition is

$$p(\nu,t) + b(t) - c = \frac{1}{\sigma}\left[p(\nu,t) + \frac{\widetilde{p}_e(t)}{A_e(\nu,t)}\right].$$

The subsidy rule

$$b(t) = \frac{1}{\sigma}\left[c + \frac{\widetilde{p}_e(t)}{A_e(t)}\right] \tag{55}$$

therefore supports $p(\nu,t) = c$ in symmetric equilibrium. Using the price first order condition, the producer's innovation condition simplifies to

$$\frac{I(t)\widetilde{p}_e(t)}{A_e(t)^2} = (1 - s_R(t))\frac{R(t)}{\theta A_e(t)}. \tag{56}$$

At the optimal price, producers capture the direct marginal benefit of improving energy efficiency. The remaining difference from the planner's condition is the benefit of reducing future research costs.

Research subsidy. In steady state, $\phi = A_e/\delta_A$ and $\lambda^\phi/\lambda^C = R/[\phi(\rho+\delta_A)]$. The planner's innovation condition becomes

$$\frac{I\widetilde{p}_e}{A_e^2} + \frac{R}{\phi(\rho+\delta_A)} = \frac{R}{\theta A_e}.$$

Substituting for research expenditure and knowledge, the planner and decentralized steady state conditions imply, respectively,

$$A_e^{\mathrm{SP}} = \left[\frac{\theta I\widetilde{p}_e}{\gamma\delta_A(1 - \theta\delta_A/(\rho+\delta_A))}\right]^\theta, \qquad A_e^{\mathrm{CE}} = \left[\frac{\theta I\widetilde{p}_e}{(1-s_R)\gamma\delta_A}\right]^\theta.$$

These conditions coincide when

$$\bar{s}_R = \frac{\theta \delta_A}{\rho + \delta_A} = \frac{0.35 \times 0.05}{0.02 + 0.05} = 0.25. \tag{57}$$

With the other distortions corrected, this subsidy eliminates the steady state innovation wedge.

Along the transition, the subsidy that reproduces the planner's innovation condition would generally vary over time,

$$s_R^*(t) = \frac{\theta A_e(t) \lambda^{\phi}(t)}{\lambda^C(t) R(t)}.$$

Since this subsidy is not available in closed form, I instead fix it at the steady state rate $\bar{s}_R = 0.25$. Innovation then satisfies

$$A_e(t) = \left[\frac{\theta \phi(t) I(t) \widetilde{p}_e(t)}{(1 - \bar{s}_R)\gamma} \right]^{\theta/(1+\theta)}. \tag{58}$$

The quantitative policy therefore combines a carbon tax equal to the social cost of carbon, the constant markup correction rate, and the steady state research subsidy. Both the business as usual economy and the carbon tax economy receive the two subsidies. These instruments align replacement incentives conditional on technology, but the fixed research subsidy does not generally reproduce the planner's innovation choice at every date. The carbon tax is set equal to the social cost of carbon without considering further benefits in the form of uninternalized innovation spillovers.

B.4.1 Climate Dynamics and Welfare

With permanent and decaying carbon stocks, the response to a unit of emissions is

$$G(v) = \varphi_L + (1 - \varphi_L)\varphi_0 e^{-\varphi_S v}.$$

Substituting this expression into equation (25) gives the two components of the social cost of carbon. The common exogenous emissions component enters atmospheric carbon but is held constant across counterfactuals.

The baseline parameter is $\psi = 0.00011$. The convex specification uses $\psi_2 = 1.29189 \times 10^{-7}$ in the model's carbon stock units to roughly match the $t = 0$ social cost carbon across counterfactuals. The reported consumption equivalent welfare gain is

$$100 \left\{ \exp \left[\frac{\int_0^{500} e^{-\rho t} \log\big(C^{\text{tax}}(t)/C^{\text{BAU}}(t)\big)\, dt}{\int_0^{500} e^{-\rho t}\, dt} \right] - 1 \right\}.$$

B.5 Derivations and Calibration Neoclassical Growth Model

Cobb–Douglas in $K - E$. Manufacturing output is produced before combining it with services:

$$X_m(t) = B_m \mathcal{E}_m(t)^{\alpha_{me}} \left[K_m(t)^{\alpha_{KL}} L_m(t)^{1-\alpha_{KL}} \right]^{1-\alpha_{me}}. \tag{59}$$

B_m is a fixed normalization matching initial manufacturing output, and K_m retains the main model's CES aggregate of durable and nondurable capital. The electricity block combines fossil fuel capital and fuel through $\widetilde{K}_e(f,t) = \mathcal{E}_e(t)^{\alpha_{ee}} K_e(f,t)^{1-\alpha_{ee}}$, retaining the remaining clean and dirty electricity and final output nests. Energy saving technology is fixed.

The calibrated exponents are $\alpha_{me} = 0.05030$, $\alpha_{KL} = 0.47352$, and $\alpha_{ee} = 0.56$. The conditional capital labor exponent differs from the main model's $\alpha = 0.5$ because fuel is now an explicit input. In particular,

$$(1 - \alpha_{KL})(1 - \alpha_{me}) = 1 - \alpha,$$

which preserves the manufacturing labor share. The calibration jointly matches capital stocks, aggregate manufacturing and electricity fuel demand, labor allocation, GDP, and the relevant factor and sectoral revenue shares. Depreciation rates equal the main model's implied rates: 5.8% and 8.3% in manufacturing, and 5% and 10% in fossil and clean electricity. Investment costs satisfy the steady state user cost conditions at those stocks. Investment expenditure and consumption are outcomes rather than additional matching targets.

Capital supply and input demand. Households own capital, with $\dot{K}_i = I_i - \delta_i K_i$. Investment is reversible, labor moves across sectors, and savings satisfy the consumption Euler equation (4). With R_i denoting the gross return to capital and c_i the conversion cost from final output into capital, no arbitrage requires $r(t) = R_i(t)/c_i - \delta_i$. In Cobb–Douglas, capital and fuel demand satisfy

$$\begin{aligned} \frac{\partial \widetilde{Y}(t)}{\partial K_i(t)} &= R_i(t), \qquad i \in \{d, nd, f, g\}, \\ \frac{\partial \widetilde{Y}(t)}{\partial \mathcal{E}_j(t)} &= p_{e,j}(t) + \chi_j \Lambda(t), \qquad j \in \{m, e\}. \end{aligned}$$

The two fuel inputs are aggregate manufacturing fuel use $\mathcal{E}_m(t)$ and fossil electricity fuel use $\mathcal{E}_e(t)$. Here $p_{e,m}(t)$ and χ_m are the common fuel price and carbon emissions coefficient for the two manufacturing sectors, while $p_{e,e}(t)$ and χ_e are their counterparts in fossil electricity generation. Fuel use can therefore fall immediately even though capital stocks are predetermined. In the Leontief case, the gross marginal product of capital instead equals its rental cost plus fuel costs per unit of capital. These benchmarks change both technological substitution and capital adjustment institutions. The within vintage counterfactuals in the preceding analysis isolate replacement and innovation more directly.

C Numerical Methods

I use Chebyshev approximations for aggregate paths and finite differences for replacement decisions and the distribution of machines. Given aggregate paths, I solve the HJB backward from a terminal value function and the distribution forward from its initial condition. This adapts the approach of Achdou et al. (2022) to capital depreciation and replacement.

Time and capital grids. I represent aggregate paths at 19 Chebyshev nodes on a time interval of 1000 years and use the associated differentiation matrix D to approximate aggregate time derivatives (Trefethen, 2000). Schesch (2024) discusses the use of Chebyshev approximations in continuous time heterogeneous agent models. Aggregate paths are held constant from the ninth node, approximately year 413.18, and the corresponding rows of D are set to zero. The terminal value function is obtained from the steady state under the terminal prices and policies.

Within each manufacturing sector, capital is normalized by the size of a new machine. I use 40 equally spaced points between 10^{-5} and one, with spacing Δk. The small positive lower endpoint approximates zero capital. Depreciation outflow is set to zero at this endpoint, while firms remain able to replace.

The HJB is solved on a finer time grid that contains all aggregate dates before the terminal closure. The maximum step is 0.25 years through year 30, 0.5 years through year 120 and 2.5 years thereafter. Aggregate paths entering the HJB are interpolated linearly. The forward distribution uses additional time steps of at most 0.05 years, shortened further when required to preserve nonnegative densities.

Solution of the HJB. Let $V_j^n = V(k_j, t_n)$ denote the value function on the finer time grid and $h_n = t_{n+1} - t_n$. Since capital drifts downward, I approximate V_k using the backward difference $(V_j^n - V_{j-1}^n)/\Delta k$. This is the upwind discretization appropriate for the HJB with negative capital drift (Achdou et al., 2022). Given the value function at t_{n+1}, the implicit step solves

$$\left(r_n + \frac{1}{h_n}\right) V_j^n + \frac{\delta k_j}{\Delta k}(V_j^n - V_{j-1}^n) - \lambda(\mathcal{V}_j^n - V_j^n) = \pi_j^n + \frac{V_j^{n+1}}{h_n}. \tag{60}$$

The depreciation term is omitted at the first capital point. At the upper endpoint, V_J^n is the value of a new machine. The replacement value uses the same date's prices and value function, so the system is nonlinear.

The logistic distribution of replacement costs makes the expected gain from replacement smooth, allowing me to solve the HJB using damped Newton iterations. I proceed backward from the terminal value function, using the next date's solution as the initial guess and requiring the maximum residual at each step to be below 10^{-10}.

Solution of the distribution. Given replacement probabilities from the HJB, I solve equation (15) forward from the pre shock stationary distribution. I use the same capital grid as for the value function. Depreciation moves firms toward lower capital, while firms that replace return to $\bar{k}$. The resulting system is linear in $\mu(k, t)$. I solve it using an implicit midpoint method, interpolating replacement probabilities between dates. Time steps are at most 0.05 years and are shortened when necessary to keep the distribution nonnegative. The discretization preserves the total mass of firms. I then use the distribution and firms' utilization decisions to compute aggregate capital, investment and fuel use.

Research productivity and lifetime fuel costs. Given a path for $A_e(t)$, I solve $\dot{\phi} = A_e - \delta_A \phi$ using the Chebyshev differentiation matrix, starting from the pre shock steady state and imposing $\delta_A \phi = A_e$ in the stationary tail. I compute lifetime fuel costs using Simpson quadrature along the machine's aging path,

interpolating replacement probabilities over age and time. As in the purchase problem, this calculation assumes full utilization until replacement.

Solution of the equilibrium. I guess paths for new machine efficiency and discounted fuel costs in the two manufacturing sectors, four capital service prices, fossil and clean electricity capital, and consumption. Given these eleven paths, the algorithm proceeds as follows.

1. Use the consumption Euler equation to compute $r(t) = \rho + \dot{C}(t)/C(t)$. Solve for research productivity and expenditure given new machine efficiency.
2. Compute purchase costs and operating profits in manufacturing. Solve the HJB backward and the distribution forward in each sector to obtain replacement, investment and the distribution of machines.
3. Solve the static allocation of labor and capital utilization. These choices determine output, fuel demand and implied capital service prices. Electricity capital follows its accumulation equations and user cost conditions, rather than a vintage replacement problem.
4. Update discounted lifetime fuel costs and the innovation conditions. Compute consumption from the resource constraint, including investment, research, fuel expenditure and realized replacement cost shocks. Taxes and subsidies enter as transfers and wedges in private decisions, with full resource costs retained.

I solve for consistent aggregate paths using damped Anderson acceleration followed, when needed, by a Levenberg–Marquardt solver. Positive paths are represented in logs. Solutions to smaller shocks provide starting values for larger shocks. The maximum scaled equilibrium residual must be below 10^{-5}. This check is separate from grid accuracy.

Climate and policy paths. For exercises with climate damages, I update atmospheric carbon using the emissions implied by the equilibrium. When the carbon tax equals the social cost of carbon, I update it using output, consumption and marginal damages. I repeat these steps with damping and record both equilibrium residuals and changes in the climate and tax paths.

The numerical closure fixes the damage multiplier and marginal damages from year 250, the seventh aggregate node. The damage multiplier is normalized to one initially and is held at its minimum after that minimum is reached. The social cost of carbon calculation retains the permanent and decaying effects of an additional emission. Its integrals use trapezoidal quadrature, with an analytical tail beyond year 1000 that holds output, marginal utility and marginal damages at their terminal values.

D Additional Quantitative Results

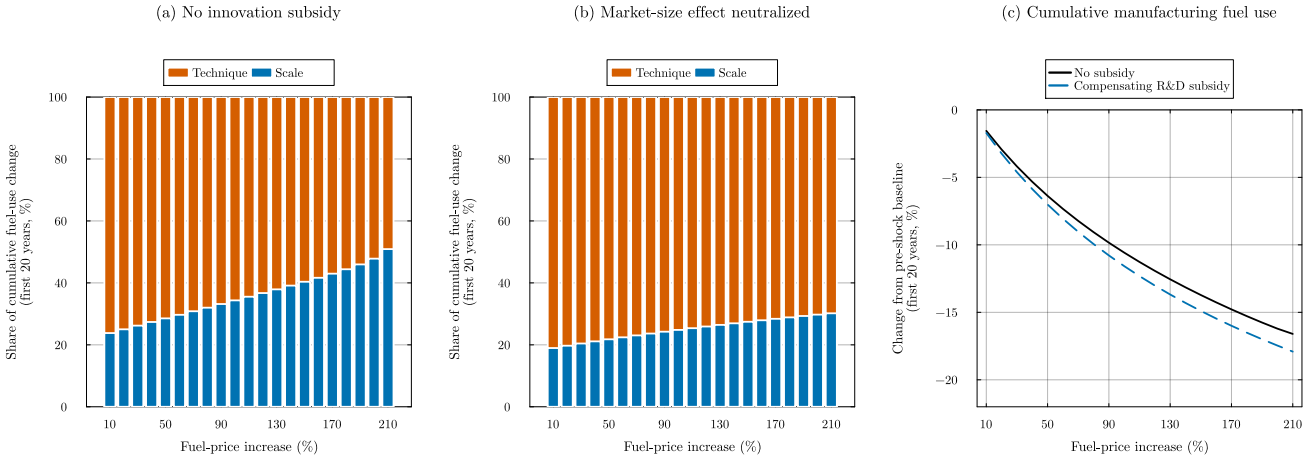


Figure D9: Market Size Effects and the Sources of Decarbonization

Notes. The figure compares immediate, permanent fossil fuel price increases of 10–210%. Panels (a) and (b) report scale and technique contributions to the change in cumulative manufacturing fuel use over the first 20 years, normalized to sum to 100%. Panel (c) reports $100[\int_0^{20} E_m(t)\,dt/(20E_m(0)) - 1]$, where $E_m(t)$ sums fuel use across manufacturing sectors. Scale measures changes in capital at initial efficiency, while technique measures the contribution of adoption and innovation. The compensating research subsidy is chosen to neutralize the effect of the investment decline on innovation incentives.

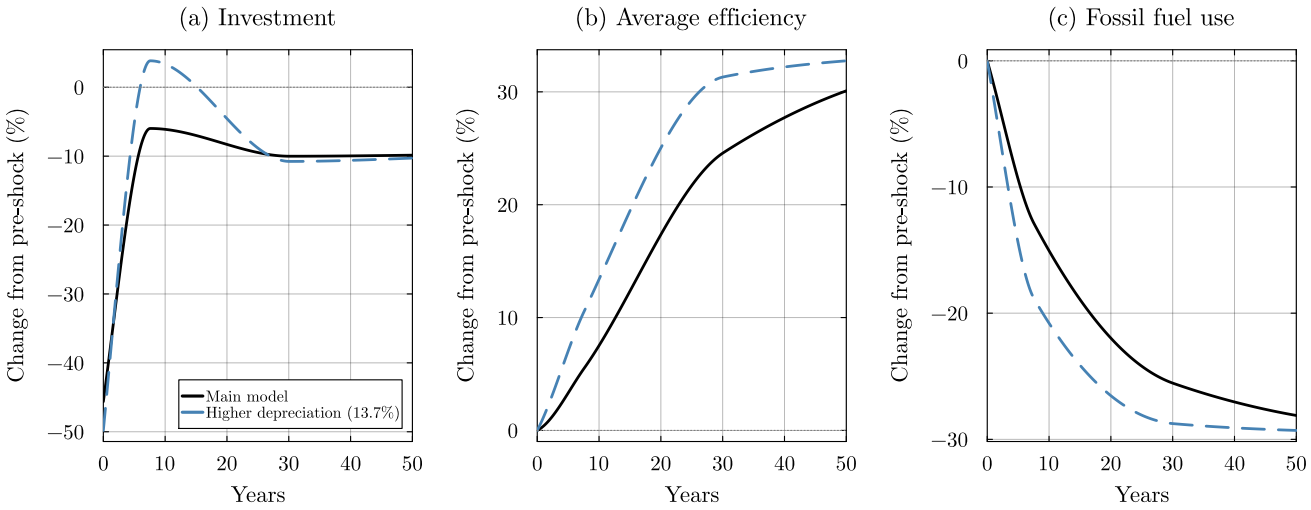


Figure D10: Manufacturing Adjustment with Higher Depreciation

Notes. The figure compares manufacturing aggregates after a permanent 150% increase in fossil fuel prices. The solid black line is the main model. The dashed blue line has 13.7% implied depreciation in both manufacturing sectors. Average energy efficiency is the capital share weighted mean of sectoral energy efficiency, with weights fixed at the pre shock level.

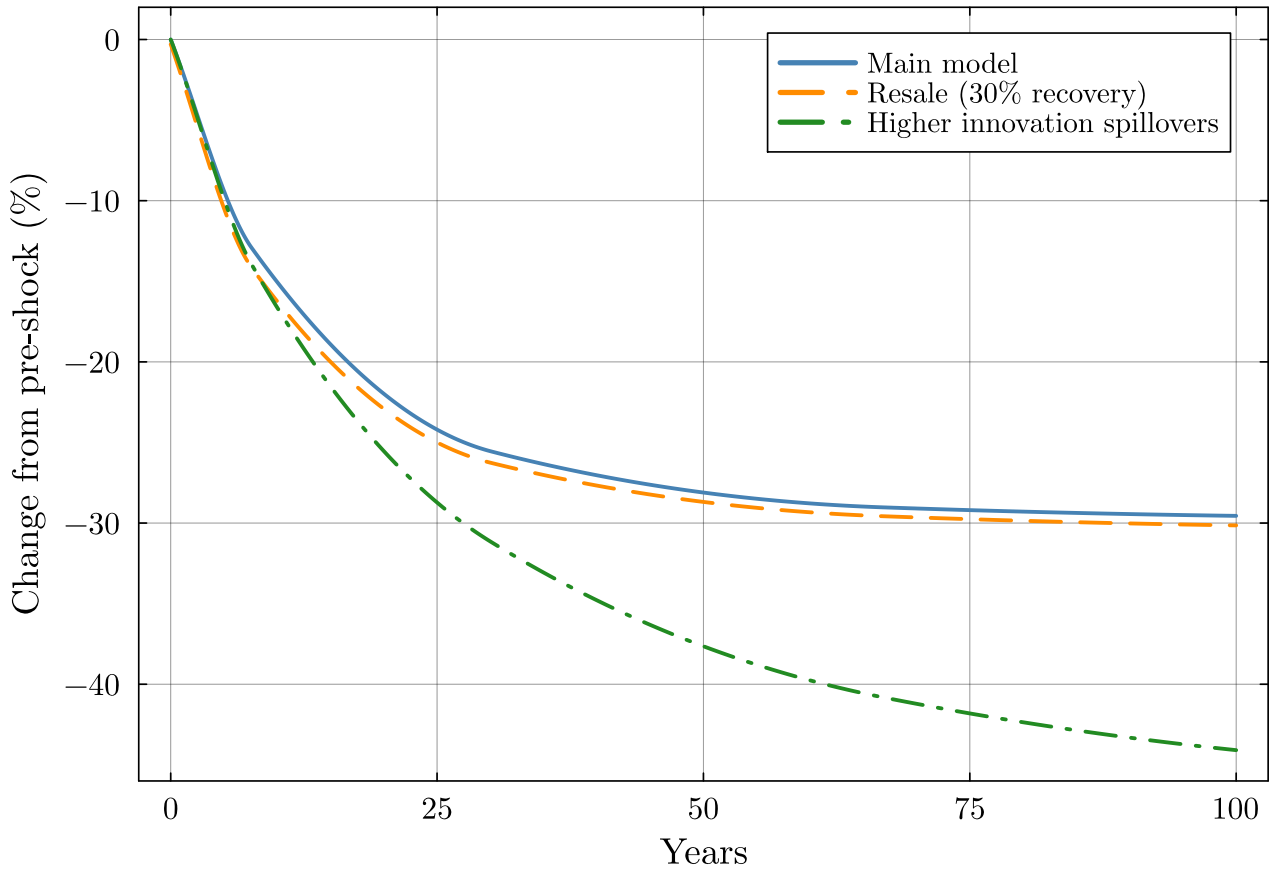
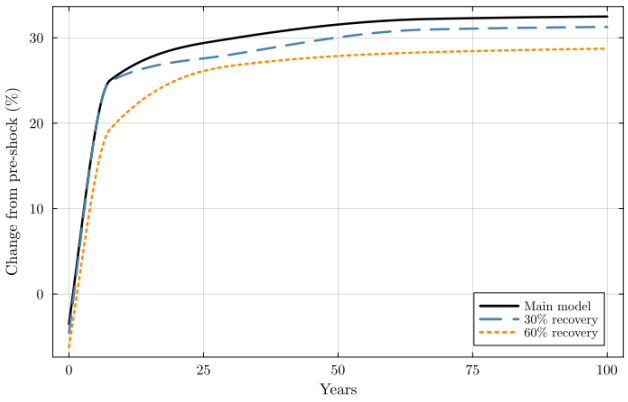
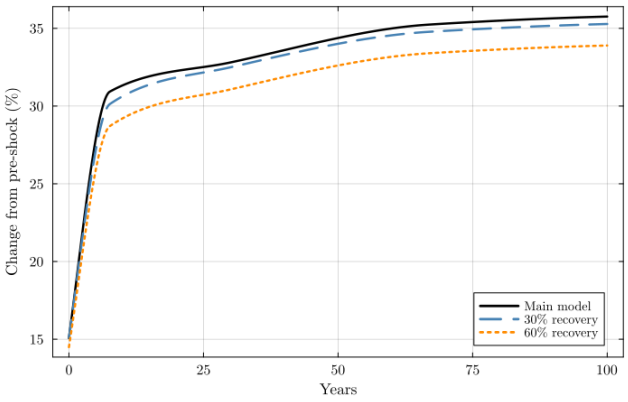


Figure D11: Manufacturing Decarbonization with Resale and Higher Innovation Spillovers

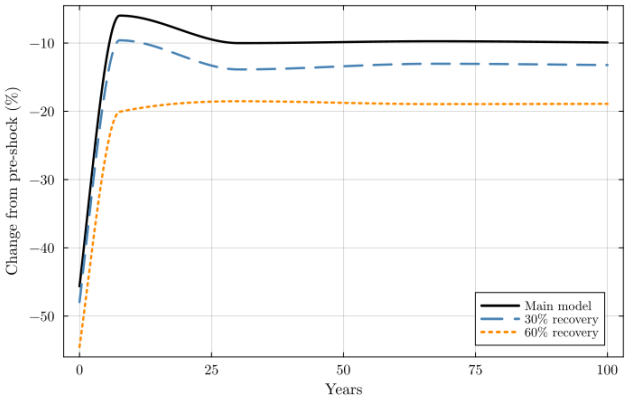
Notes. The figure compares manufacturing fossil fuel use in the main model, the resale calibration with a 30 percent recovery rate, and the calibration with higher innovation spillovers after a permanent 150 percent increase in fossil fuel prices. The y axis shows percentage changes in fuel use relative to each economy's pre shock steady state. Resale includes fuel use by both incumbents and fringe firms. The figure shows the first 100 years. The simulations abstract from climate damages.



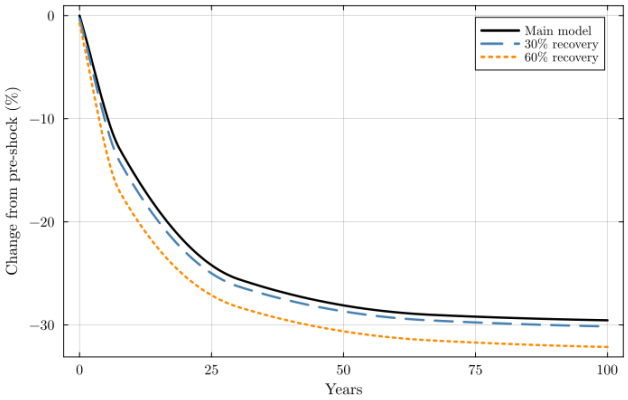
(a) Energy efficiency of new machines $A_e(t)$: durable manufacturing



(b) Energy efficiency of new machines $A_e(t)$: less durable manufacturing



(c) Manufacturing investment



(d) Manufacturing fuel use

Figure D12: Resale and Transition Dynamics After a Permanent Energy Price Increase

Notes. The figure compares the main model without resale with the separately calibrated economies targeting recovery rates of 30 and 60 percent. The shock permanently raises fossil fuel prices by 150 percent. All responses are percentage changes from each economy's pre shock steady state. Panels (a) and (b) show the energy efficiency of new machines. Panel (c) measures aggregate manufacturing investment in new capacity. Used machine transactions are excluded. Panel (d) includes fuel use by both incumbents and fringe firms. The simulations abstract from climate damages.

D.1 Larger Spillovers

To examine whether slow adoption remains important when innovation generates greater long run substitution away from fossil fuels, I increase the strength of knowledge spillovers while retaining the baseline research curvature $\theta = 0.35$. Under the baseline law of motion in equation (22), steady state research productivity satisfies $\phi = A_e/\delta_A$. Combining this expression with equation (20) gives

$$A_e = \left(\frac{\theta M \bar{k} \tilde{p}_e}{\gamma \delta_A} \right)^{\theta}.$$

Holding replacement demand and the discounting and survival terms entering lifetime energy costs fixed, the steady state elasticity of new machine energy efficiency to a permanent increase in energy prices is therefore θ.

I modify the law of motion for research productivity to

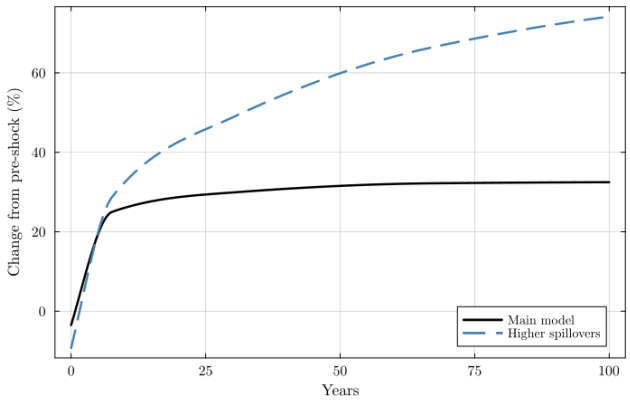
$$\dot{\phi}(t) = -\delta_A \phi(t) + A_e(t)^x.$$

In steady state, $\phi = A_e^x/\delta_A$, so equilibrium new machine efficiency satisfies

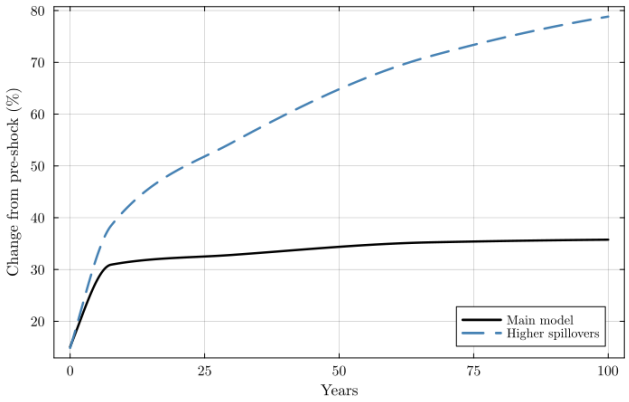
$$A_e = \left(\frac{\theta M \bar{k} \tilde{p}_e}{\gamma \delta_A} \right)^{\frac{1}{1+1/\theta - x}}.$$

Setting $x = 1 + 1/(2\theta)$ doubles the partial equilibrium elasticity from θ to 2θ, or from 0.35 to 0.70. This changes the accumulation of research productivity without changing the firm's innovation problem conditional on $\phi(t)$. I recalibrate the economy to the same steady state target moments as in the baseline.

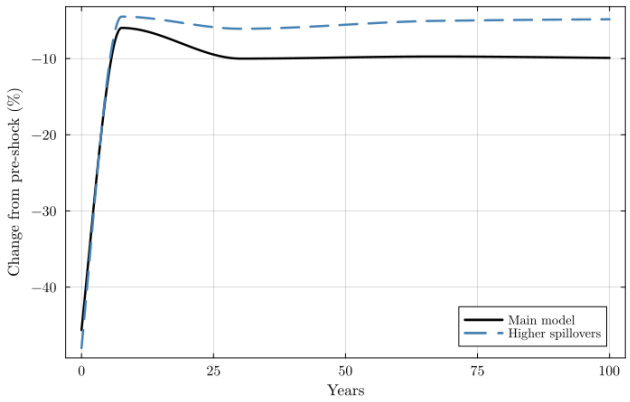
Figure D13 compares the responses to a permanent 150% increase in fossil fuel prices. Larger spillovers generate substantially greater improvements in new machine efficiency, but the initial contraction in manufacturing investment remains similar. The additional fuel savings emerge gradually as firms replace existing machines. Thus, greater long run technological responsiveness does not eliminate the role of slow adoption along the transition.



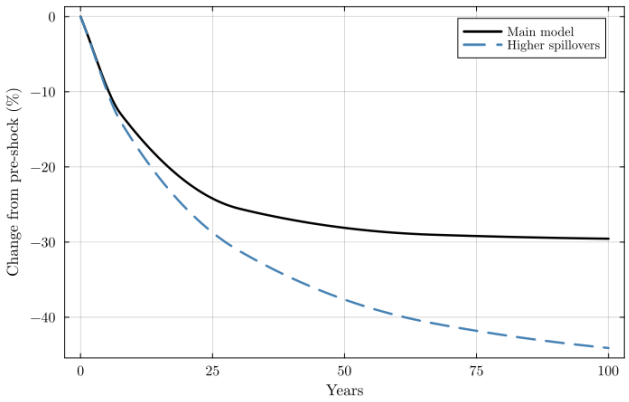
(a) Energy efficiency of new machines $A_e(t)$: durable manufacturing



(b) Energy efficiency of new machines $A_e(t)$: less durable manufacturing



(c) Manufacturing investment

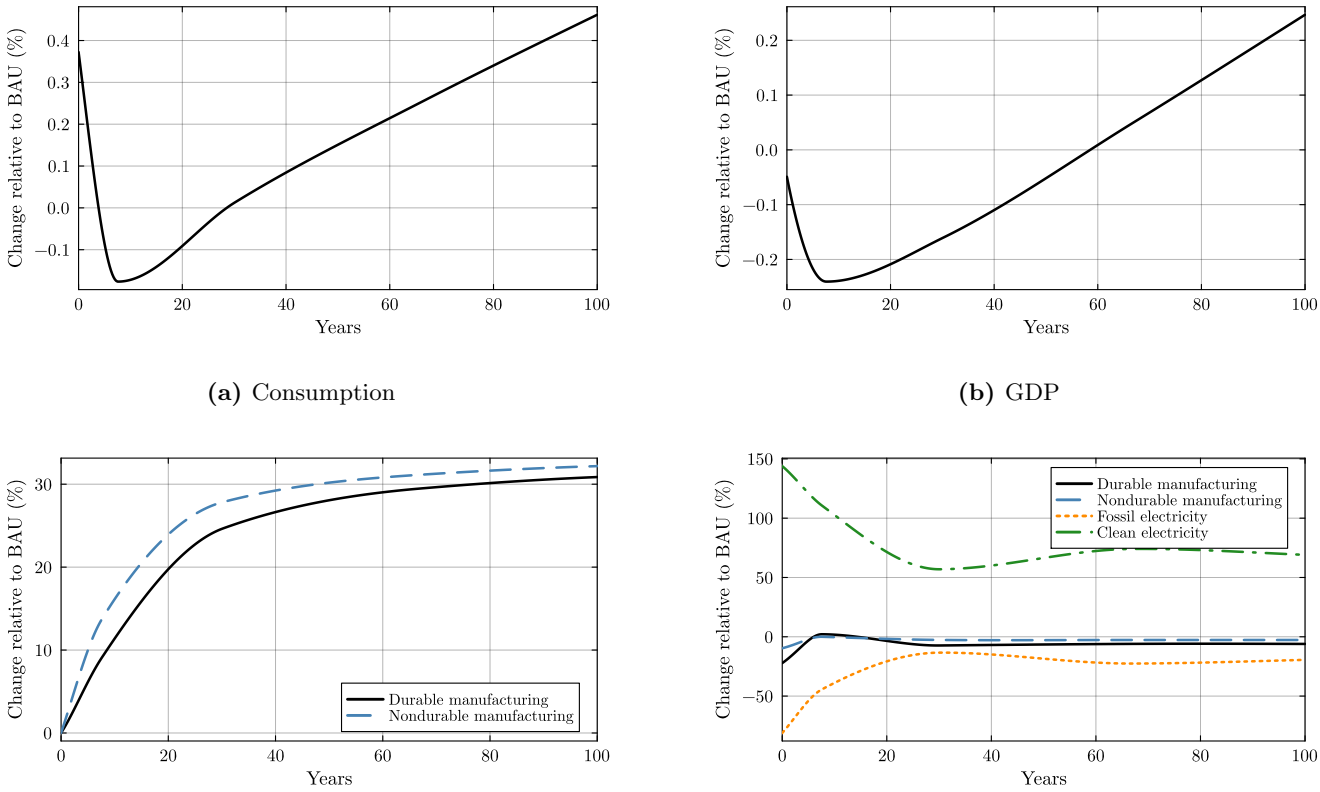


(d) Manufacturing fuel use

Figure D13: Transition Dynamics After a Permanent 150 Percent Increase in the Price of Energy

Notes. The figure compares the main model with the larger spillover recalibration after a permanent 150 percent increase in fossil fuel prices. All responses are percentage changes from each economy's pre shock steady state. Panels (a) and (b) show the energy efficiency of new machines in durable and less durable manufacturing. Panels (c) and (d) show aggregate manufacturing investment in new capacity and manufacturing fossil fuel use, respectively. The simulations abstract from climate damages.

D.2 Further Results on Carbon Policy



(a) Consumption

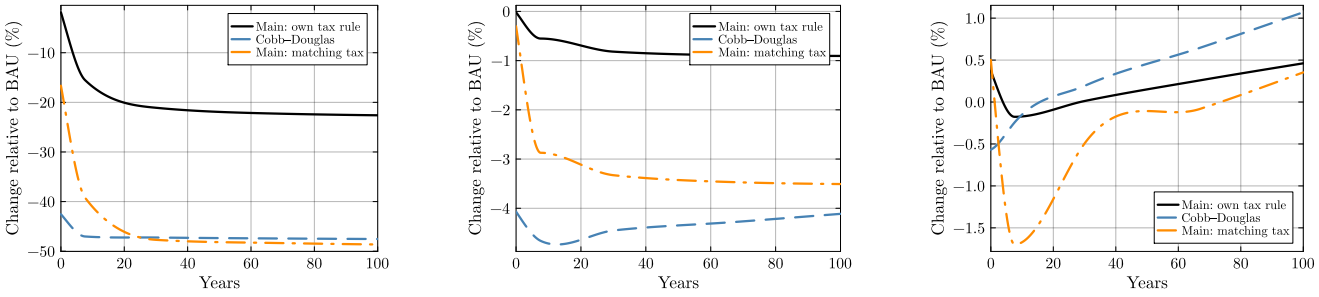
(b) GDP

(c) Average energy efficiency

(d) Investment

Figure D14: Adjustment to Carbon Pricing in the Main Model

Notes. All panels report percentage changes relative to the policy BAU under baseline damages. Both paths introduce the 25% research subsidy and full markup correction at date zero from the no policy steady state. Panel (c) reports average energy efficiency in each manufacturing sector, weighted by the distribution of machines. Panel (d) reports gross investment in capacity by sector. Carbon tax revenue is rebated through lump sum transfers.



(a) Emissions **(b)** Manufacturing output **(c)** Consumption

Figure D15: The Cost of Matching the Cobb–Douglas Emissions Reduction

Notes. The figure compares the main model and Cobb–Douglas under their own carbon tax rules with the main model under a constant tax matching the Cobb–Douglas emissions reduction at year 25. All responses are relative to the corresponding model's BAU under baseline damages. The two main model experiments share the same BAU with research and markup corrections. Emissions cover manufacturing and fossil fuel electricity generation. Manufacturing output excludes services and uses each model's calibrated manufacturing production function.

D.3 Innovation and Replacement Subsidies

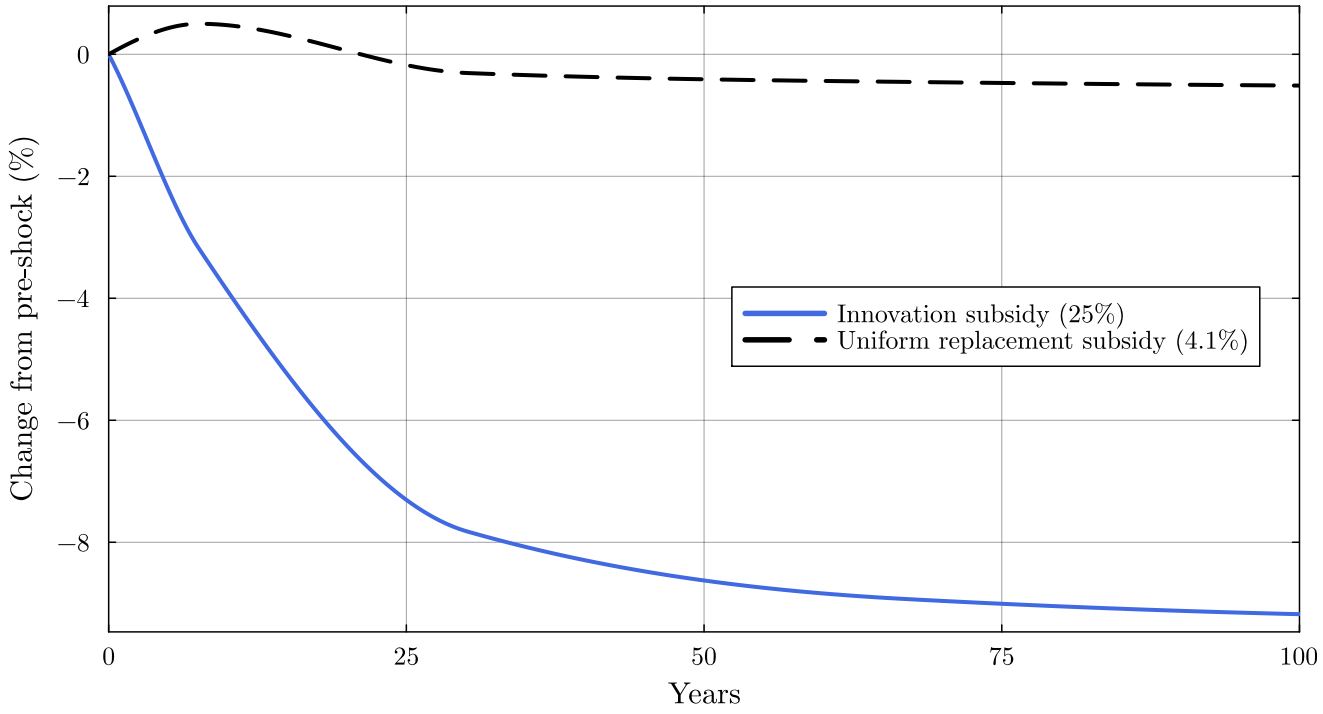


Figure D16: Manufacturing Fuel Use under Innovation and Replacement Subsidies

Notes. The figure compares permanent subsidies to research expenditure and manufacturing equipment purchases in the main model without electrification. The y axis shows percentage changes in manufacturing fossil fuel use from the common pre shock steady state. The simulations abstract from climate damages, carbon taxes and markup corrections.

E Electrification: Model and Further Results

E.1 The replacement problem

The Bellman equations and expected replacement values are given in equations (27)–(30) in the main text. Electric and fossil fuel using machines differ in purchase price and operating cost, but I abstract from additional switching costs.

Fossil fuel machines retain the production technology from Section 3. Electric machines require b units of electricity per unit of capital in use. Profits are

$$\pi_F(k,t) = \max_{u\in[0,1]} uk\left(P^K(t) - \frac{p_e(t)}{A(k,t)}\right), \tag{61}$$

$$\pi_E(k,t) = \max_{u\in[0,1]} uk\left(P^K(t) - b\mathcal{P}_e(t)\right). \tag{62}$$

Electric machines are supplied by perfectly competitive producers at a constant marginal cost. Their electricity requirement b is fixed. Producers of fossil fuel machines continue to choose energy efficiency through the innovation problem in Section 3.

E.2 The distribution of machines and equipment demand

Let $\mu_F(k,t)$ and $\mu_E(k,t)$ denote the densities of firms operating the two machine types. The probabilities $P_F^{\text{rep}}(k,t)$ and $P_E^{\text{rep}}(k,t)$ refer to replacement with the same fuel. The probabilities $P_F^{\text{sw}}(k,t)$ and $P_E^{\text{sw}}(k,t)$ refer to switching to the other fuel. For $0 < k < \bar{k}$, the distributions evolve according to

$$\partial_t\mu_F(k,t) = \partial_k\big(\delta k\mu_F(k,t)\big) - \lambda\big(P_F^{\text{rep}}(k,t) + P_F^{\text{sw}}(k,t)\big)\mu_F(k,t), \tag{63}$$

$$\partial_t\mu_E(k,t) = \partial_k\big(\delta k\mu_E(k,t)\big) - \lambda\big(P_E^{\text{rep}}(k,t) + P_E^{\text{sw}}(k,t)\big)\mu_E(k,t). \tag{64}$$

The demand for new machines includes both replacement within a fuel type and switching from the other type,

$$M_F(t) = \lambda\int_0^{\bar{k}}\big[P_F^{\text{rep}}(k,t)\mu_F(k,t) + P_E^{\text{sw}}(k,t)\mu_E(k,t)\big]dk, \tag{65}$$

$$M_E(t) = \lambda\int_0^{\bar{k}}\big[P_E^{\text{rep}}(k,t)\mu_E(k,t) + P_F^{\text{sw}}(k,t)\mu_F(k,t)\big]dk. \tag{66}$$

The boundary conditions are

$$\delta\bar{k}\mu_F(\bar{k},t) = M_F(t), \qquad \delta\bar{k}\mu_E(\bar{k},t) = M_E(t). \tag{67}$$

Gross investment in new machines is $I_F = M_F\bar{k}$ and $I_E = M_E\bar{k}$. The market for new fossil fuel machines, M_F, determines the demand facing their producers and hence innovation incentives. Both replacement and switching end the operating life of an existing fossil fuel machine, i.e. there is no retrofitting.

Capital services within a manufacturing sector are the sum of fossil fuel and electric capital in use,

$$K^u(t) = K^u_F(t) + K^u_E(t) = \int_0^{\bar{k}} u_F(k,t)k\mu_F(k,t)dk + \int_0^{\bar{k}} u_E(k,t)k\mu_E(k,t)dk. \tag{68}$$

The aggregation across the two manufacturing sectors remains as in the main model. Manufacturing fossil fuel demand and electricity purchases are

$$\mathcal{E}_F(t) = \int_0^{\bar{k}} \frac{u_F(k,t)k}{A(k,t)}\mu_F(k,t)dk, \qquad D_e(t) = bK^u_E(t). \tag{69}$$

Thus, fuel use can fall through a smaller capital stock, better embodied efficiency, or a shift toward electric machinery.

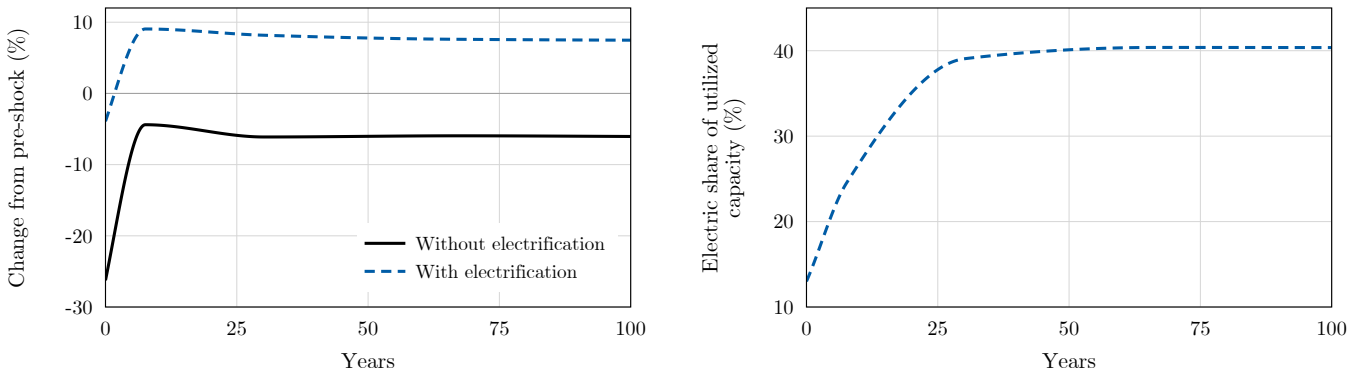
E.3 Calibration and the fuel switching response

In the main extension, manufacturing purchases electricity at a fixed import price. I choose $b\mathcal{P}_e$ to equal the operating cost per unit of fossil fuel capital in use in the initial steady state. I calibrate P_e in each manufacturing sector so that electricity accounts for 13% of initial energy expenditure. The calibration retains the macroeconomic targets of the main model, including implied depreciation rates of 5.8% and 8.3% in the two manufacturing sectors. The electric machine prices and remaining calibrated parameters are jointly determined to match these targets.

Kaartinen and Prane (2024) study fuel switching among Swedish manufacturing firms. Their matched event study shows an increase of about ten percentage points in electricity's share of energy use between 2015 and 2020 as carbon taxes rise. Under the permanent tax used in Figure 8, the electric share of capital in use rises from 13% to 21.1% after five years. The response is similar in magnitude over this horizon, similar to their estimates.

Figure E1 reports the investment and adoption paths. Manufacturing investment falls by about 26% on impact in the main model and by 4% in the electrification extension. By year 25, manufacturing investment in the extension exceeds its pre shock level. Carbon taxes still discourage investment in fossil fuel machines, but they make electric machinery more attractive. The composition of equipment purchases therefore matters for the aggregate investment response.[27]

[27]This mechanism allows carbon pricing to stimulate investment in cleaner capital even when it reduces investment in fossil fuel machines. It is consistent with the finding in Brown et al. (2026) that high emission Swedish firms increase both total capital investment and the share devoted to abatement in response to carbon pricing.



(a) Manufacturing investment

(b) Electric share of utilized manufacturing capacity

Figure E1: Investment and Adoption under the Same Carbon Tax

Notes. The tax is \$40/tCO$_2$ in 2010 dollars and applies to manufacturing and fossil fuel electricity generation. Panel (a) reports real manufacturing investment expenditure relative to each economy's pre shock level. Panel (b) reports electric utilized capacity divided by total manufacturing utilized capacity in the imported electricity extension. Climate damages and research and markup corrections are absent.

E.4 The size of the subsidy and policy comparisons

I use the fiscal cost cited in Curtis et al. (2023) to choose the scale of the uniform replacement subsidy. The estimated cost of bonus depreciation is \$285 billion over 2019–2028. Relative to 2019 US GDP of \$21.5 trillion, this gives a ten year budget of about 1.3% of initial GDP. I allocate this budget to manufacturing as an illustrative policy and approximate its cost through a constant subsidy to the purchase price of new machines. The calibrated subsidy $s = 0.041$ implies fiscal costs of 1.3% of initial GDP over ten years, close to this benchmark.

For the targeted policy, I solve the electrification equilibrium and choose s_e so that

$$\int_0^{10} \sum_{i\in\{d,nd\}} s_e P_{e,i}(t) I_{E,i}(t) dt = \int_0^{10} \sum_{i\in\{d,nd\}} s P_i(t) I_i(t) dt. \tag{70}$$

The right hand side is evaluated in the main model under the uniform subsidy and the left hand side in the electrification extension. The resulting electric machine subsidy is 11.3%. The two budgets differ by less than 0.03%. Both policies are permanent, although expenditure is matched over the first ten years without discounting.

The constant tax and equal budget subsidy experiments abstract from climate damages and research and markup corrections. Each path is measured relative to its own pre shock steady state. Uniform subsidies leave fuel use unchanged after 25 years due to offseting scale and innovation effects. Over the first 25 years, cumulative manufacturing fuel use rises by about 0.25% under the uniform subsidy and falls by 8.4% under the targeted subsidy.

Table 7 instead includes evolving climate damages to evaluate welfare effects. Both policies are compared with the same BAU transition without a research subsidy or markup correction. Table 7 reports

the difference between the tax and subsidy consumption equivalents, in percentage points relative to this common BAU. The imported electricity specification does not account for the emissions embodied in imported electricity. The domestic extension shows how results change when these emissions are accounted for explicitly.

E.5 Domestic electricity supply and emissions

I next allow manufacturing firms to purchase domestically produced electricity. Let Q_e denote total generation and X_e electricity used in the final output production function. Market clearing requires

$$Q_e(t) = X_e(t) + \sum_{i \in \{d,nd\}} b_i K^u_{E,i}(t). \tag{71}$$

The electricity price $\mathcal{P}_e(t)$ equals its marginal product in final production. I recalibrate the domestic economy to retain the initial manufacturing electricity expenditure shares. In the carbon tax experiments, manufacturing and fossil fuel electricity generation pay the same carbon price per tonne of CO_2.

Let χ_i convert sectoral fuel use into CO_2. Direct manufacturing emissions and electricity sector emissions are

$$Z_m = \sum_{i \in \{d,nd\}} \chi_i \mathcal{E}_{F,i}, \qquad Z_e = \chi_e \mathcal{E}_e, \qquad Z^{\text{model}} = Z_m + Z_e. \tag{72}$$

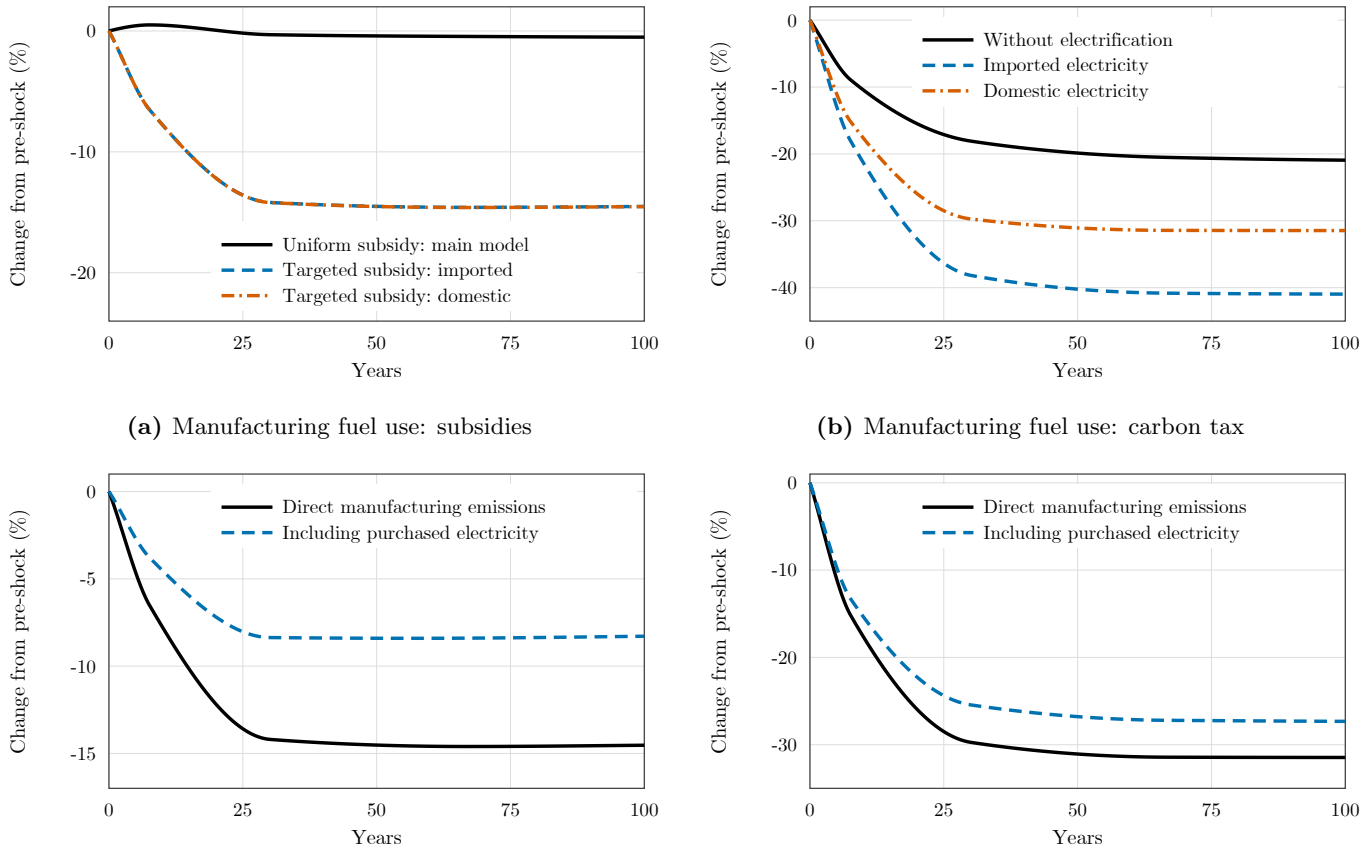
I allocate emissions embodied in electricity purchased by manufacturing using the average grid intensity,

$$Z_m^{S2} = \frac{\sum_i b_i K^u_{E,i}}{Q_e} Z_e, \qquad Z_m^{S1+S2} = Z_m + Z_m^{S2}. \tag{73}$$

Figure E2 compares the transitions. Under the targeted subsidy, manufacturing fuel use falls by 13.6% after 25 years in both electricity supply arrangements. Electric manufacturing machines initially account for only 4% of electricity consumption. Their additional demand raises electricity supply by 5.3%, with almost no increase in the electricity price. Manufacturing emissions including purchased electricity fall by 8.0%, and total modeled emissions fall by 4.2%. The increase in electricity sector emissions offsets a fraction of direct manufacturing emissions savings equal to

$$\frac{Z_e(t) - Z_e(0)}{Z_m(0) - Z_m(t)}, \tag{74}$$

which is 39% at year 25. Under the carbon tax, the electricity price rises by about 25% at year 25. Manufacturing fuel use falls by 28.5%, compared with 36.3% with imported electricity and 17.1% without electrification. Manufacturing emissions including purchased electricity decline by 24.4%, and total modeled emissions decline by 19.9%.



(a) Manufacturing fuel use: subsidies

(b) Manufacturing fuel use: carbon tax

(c) Manufacturing emissions: domestic subsidy

(d) Manufacturing emissions: domestic carbon tax

Figure E2: Electricity Supply and the Emissions Consequences of Electrification

Notes. This figure shows the carbon price and subsidy implications of electrification depending on how additional electricity is supplied. Paths are relative to each economy's pre shock steady state, without climate damages or research and markup corrections. The tax is \$41/tCO_2, while subsidies are chosen to match uniform subsidy, main model outlays over ten years. Lower panels distinguish direct manufacturing emissions from direct emissions plus those allocated to purchased electricity.

Finally, Table E1 repeats the matched fuel reduction exercise with domestic electricity. Subsidies continue to yield lower welfare than carbon taxes. At the 20% manufacturing target, the tax reduces total modeled emissions by 14.3%, compared with 6.2% under the subsidy. The tax prices fossil fuel use in generation as well as manufacturing, whereas the subsidy expands electricity demand without taxing generation emissions. Unlike the imported matched target exercise in Table 7, the domestic carbon tax covers manufacturing and fossil fuel electricity generation. Differences between the two tables therefore reflect tax coverage as well as electricity supply.

Table E1: Matched Manufacturing Fuel Reductions with Domestic Electricity

Fuel reduction at year 25	Carbon tax (2010 \$/tCO$_2$)	Electrification subsidy (%)	Welfare gap (tax minus subsidy, pp)
10%	12.4	9.0	0.142
20%	27.3	14.5	0.268
30%	42.5	18.7	0.357

Notes. Targets refer to manufacturing fossil fuel reductions at year 25 relative to the same domestic climate BAU without research or markup corrections. The carbon tax covers manufacturing and fossil fuel electricity generation. Subsidies reimburse electric manufacturing machine purchases. The welfare gap is the tax consumption equivalent change minus the subsidy consumption equivalent change, in percentage points. Welfare includes climate damages over 500 years.